CHAPTER 3

WORK AND ENERGY

CHAPTER'S OBJECTIVES

- To define and calculate work done on objects
- To understand and use the concepts of kinetic and potential energy
- To understand and calculate the concept of power
- To understand and use the law of conservation of energy
- To analyze the function and definition of simple machines and their mechanical advantage

Energy is the most important concept in science. Its importance comes from fact that it is a scalar quantity and therefore it does not depend on direction.

Energy is closely related to force and motion. Any type of motion requires energy. For example, lifting an object from the ground and place it on the table top requires energy. Pushing or pulling objects require some sort of energy.

Sources of energy are either moving matter like wind , water, and air, which produce forces and cause motion. Falling water from high altitude can turns a turbine blades to produce electricity and the energy captured from the wind by the sail propels the. Food and gasoline are just two examples of stored energy sources that cause motion as well.

Force and motion yield work, another important concept in science, but not as fundamental as energy. In this chapter we will discuss the tools and means of transferring energy from a source to a system.

3.1 Work (W)

To do work in a scientific meaning, a force F must be applied on an object to move it a distance d. the force and distance traveled must be parallel to each other. This formal definition of work implies the following

- Forces perpendicular to distance do not do work. For example, the force of earth on the moon does not do work

- Work is zero if $d = 0$, or $F = 0$

- Work done by friction force is negative

Build on this definition, work can be written as

$$W = F \times d \tag{3.1}$$

Work is a scalar quantity and, thus, has a magnitude only. The metric (SI) unit of work is Newton times meter (N.m) or Joule (*J*). 1 *J* of work, is the work resulted from moving an object a 1 meter by a 1 Newton force. Figure 3.1 illustrates the definition of work.

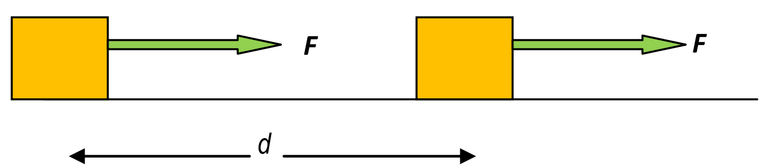
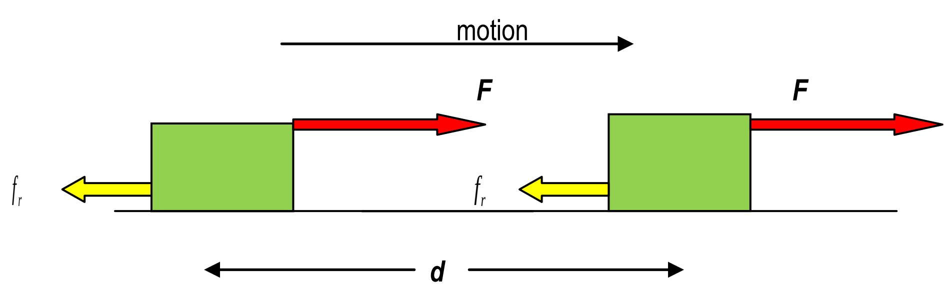


Figure 3.1 Force *F* is doing work on an object through a distance *d*.

Example 3.1

A 50 kg box is pulled 20 m a long a horizontal floor by a constant force of 200 N exerted by a person. The floor is rough and provides a friction force of 50 N. Determine the work done by (a) the person and (b) by the friction force, and the net work

Solution (1)



Given Data

$$F = 200N,\ f_r = 50N$$
$$d = 20m,\quad W_F,\ W_{f_r}\ ?$$

$$\because W = F \times d,$$
$$\therefore (a)\ W_F = (200N) \times (20m) = 4000J,\ and$$
$$(b))\ W_{f_r} = -(50N) \times (20m) = -1000J$$
$$(c) W_{net} = 4000J - 1000J = 3000J$$

Solution (2)

$$F_{net} = 200N - 50N = 150N,$$
$$W_{F_{net}} = (150N) \times (20N) = 3000J$$

3.2 Mechanical Energy

Mechanical energy is defined as the ability to do work on an object (say A) by another object (say B). This work appears as energy of motion (of object B) or stored in the object (object B). Work and energy are related, and they share the same unit of measurement, the Joule. In this chapter, we will focus on two types of mechanical energy, the potential and kinetic energy.

3.2.1 Potential Energy (PE)

Potential energy is the energy stored in an object as a result of doing work on it. Potential energy comes into different forms of energy. There is the chemical potential energy stored in the bonds of food, wood, and gasoline molecules, elastic potential energy in compressed and stretched springs, electrical potential energy in a system of electrical charges, and gravitational potential energy. Our attention will be on the gravitational potential energy.

Gravitational Potential Energy (PE_g)

Gravitational potential PE_g is the energy an object has because of its position relative to the ground. It is given as a product of an object weight and its distance or height from ground level. This can be written as

$$PE_g = weight \times h = wh = mgh \tag{3.2}$$

Unit of measurement of PE_g is Joule. The object of mass m shown in Figure 3.2 is at a height h above the ground level and therefore has PE_g.

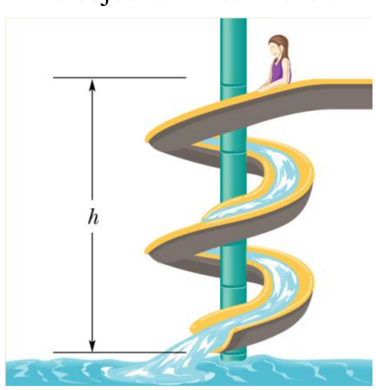


Figure 3.2: An object at height h above ground has a gravitational potential energy equal *mgh*. Credit: *Halliday, Resnick, and Walker*,"*Fundamental of physics" 6th Ed. John Willey &Sons, 2001*.

Example 3.2

A 2kg object is lifted from ground to a height of 3 m vertically above the ground as shown in Figure 3.3. Find the following

(a) The work done by the force of gravity
(b) The work done by the lifting force.
(c) The gravitational potential energy at height h. Compare with your answer at (b)

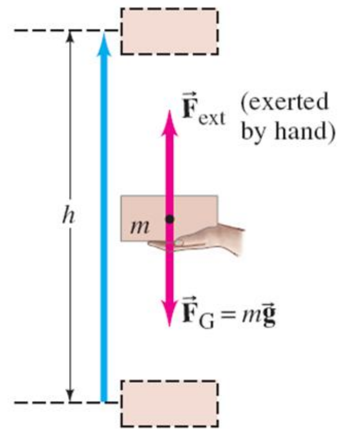


Figure 3.2: lifting an object above ground to a height h.

Solution

A free body diagram is shown in Figure 3.2

$$Given\ Data: m = 2kg,\ h = 3m$$
$$Wanted: (a)\ W_g,\ (b)\ w_{F_{ext}},\ (c) PE_g$$

$$(a)\ W_g = F \times d = -mgh = -(2kg)(9.8m/\sec^2)(3m) = -58..8\ J \quad (why\ "-")$$

$$(b)\ W_{F_{ext}} = F \times d = +mgh = +(2kg)(9.8m/\sec^2)(3m) = +58.8J \ (why\ "+")$$

$$(c)\ PE_g = weight \times height = mgh = (2kg)(9.8m/\sec^2)(3m) = 58.8\,J$$

Caution: Notice that the work done by the external force equals PE_g.

3.2.2 Kinetic Energy (*KE*)

Kinetic energy *KE* is the energy an object has because of motion. All objects in motion like a car, a planet, and atom have kinetic energy. This kinetic energy given to the car, the planet, and the atom is a result of work done by external forces acting on these objects. For example, to move a car from rest along a horizontal floor, a force is needed to increase its speed from zero to a higher speed. The work done by the force equals the car's kinetic energy.

For an object of mass *m* and velocity v its kinetic energy is given as

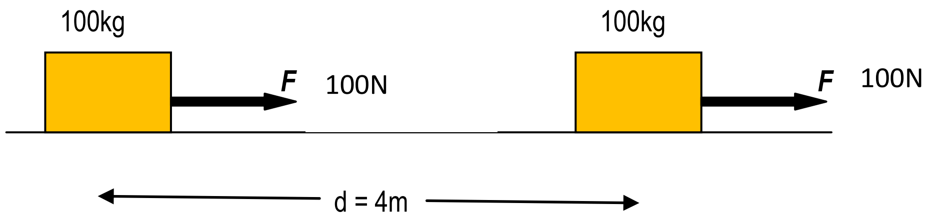
$$KE = \frac{1}{2}mv^2 \tag{3.3}$$

Example 3.3

A force of 100 N acts on a 100 kg box placed on a frictionless floor for 10 seconds. The box starts from rest and reaches a final speed of 2 m/s. During these 10 s, the box has moved 4 m. Find

(a) the work done on the box, and
(b) the final kinetic energy of the box.

Solution



Given Data

$$v_i = 0,\ m = 100kg,\ F = 50N,\ t = 10s$$
$$d = 4m,\quad W\ and\ KE\ ?$$

(a) Work done the box

$$\because W = F \times d$$
$$\therefore W = (50N) \times (4m) = 200J$$

$$(b) \because KE = \frac{1}{2}mv^2$$
$$\therefore KE = \frac{1}{2} \times (100kg) \times (2m/s)^2 = 200J$$

Caution: Notice that the work done on the object (200J) equals to its final KE.

3.3 Power (*P*)

Power P is defined as how much time it takes for a force to do work, or how fast work is done. For example, will the job of lifting 2000kg box from a ground level to the fourth floor by a mechanical system takes three minutes or a day? Will the work done by the friction force to accelerate a car at red light stop to 40 km/h done in a short or long time?

Power is, therefore, given as

$$P = \frac{work}{time} =,\ or \qquad (3.4)$$
$$P = \frac{W}{t}$$

In metric (SI) units, power is measured in joules per second or watt: 1 watt = 1J/s is the power of doing 1 J of work in 1 s. The watt is small unit and for practical purposes, kilo watt or kW is used. One kW=1000 watt. The old unit of measurement of power was the horsepower (hp) and it is still used, occasionally, to describe the power of cars engine. One horsepower is equivalent to 746 watt or 0.746 kW.

Example 3.4

An elevator with a person inside weighs 5000 N. If it is raised to a height of 15 m in 10 s. Find

(a) The work done on the elevator by the motor.
(b) The power

Solution

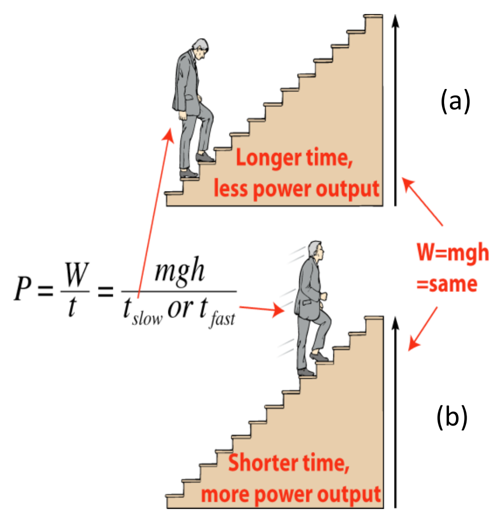
Given Data

$$F = 5000N, d = 15m, t = 10s,$$
$$wanted: \ P?$$

$$(a) \because W = F \times d$$
$$\therefore W = (5000N)(15m) = 75000J = 7.5\times10^4 J$$

$$(b) \because P = \frac{W}{t} = \frac{F\times d}{t},$$
$$\therefore P = \frac{F\times d}{t} = \frac{(5000N)\times(15m)}{10s} = 7500Nm/s = 7.5\times10^3 \ watt$$

Example 3.5 (a) A 70 kg man was advised to lose energy (for weight lose) through exercise. The man took advice from a friend of his suggesting that he should walks up a flight of stairs. Another friend suggested that he should run the flight of stairs. He walks up the flight of stairs in 10 s. In another trial, the man runs up the stairs in 4 sec as Figure3.3 shows. The vertical height of the stairs is 5m. In each case find and compare the man's power output and how much energy did this require?



Figur 3.3: Comparing power delivered by a person walking up a flight of stairs (a) and running up the stairs (b) Credit: *B.W. Tillery, E. D. Enger, and F. C. Ross, "Integrated science", 3rd Ed., McGraw Hill 2004.*

Solution

Given Data

$$m = 70,\ g = 9.8 m/s^2,\ d = h = 5m,\ t(walk) = 10s,\ t(run) = 4s)$$
$$wanted:\ \ P(walk),\ P(run)?$$

$$\because P = \frac{W}{t} = \frac{mgh}{t}$$
$$\therefore\ P(walk) = \frac{(70kg)(9.8m/s^2)(5m)}{10s} = 343watt$$
$$\therefore\ P(run) = \frac{(70kg)(9.8m/s^2)(5m)}{4s} = 857.5watt$$

Notice that the power of run is much greater than that of walk. However, the energy needed in both cases can be found as

$$Energy\ required\ E = Power \times time,$$
$$E(walk) = 343watt \times 10s = 3430J$$
$$E(run) = 857.5watt \times 4s = 3430J$$

The energy (work) spend by the man from the food he eats (stored as chemical energy) is the same in both cases. Therefore, as far as energy release is concerned the two cases are the same.

3.4 Conservation of Mechanical Energy

The mechanical energy E is a sum of kinetic and potential energies at any moment. That i

$$E = KE + PE = \frac{1}{2}mv^2 + mgd \tag{3.5}$$

When no friction or resistance to the motion is present, the energy of an object remains constant. During a process such as free fall in air, the kinetic energy and potential energy change as they transform back and forth into each other, but their sum remains constant. Mathematically, this means that the change in energy E or $\Delta E = 0$. That is

$$\Delta E = 0 = \Delta KE + \Delta PE,\ \ or$$
$$0 = (KE_2 - KE_1) + (PE_2 - PE_1) \tag{3.6}$$

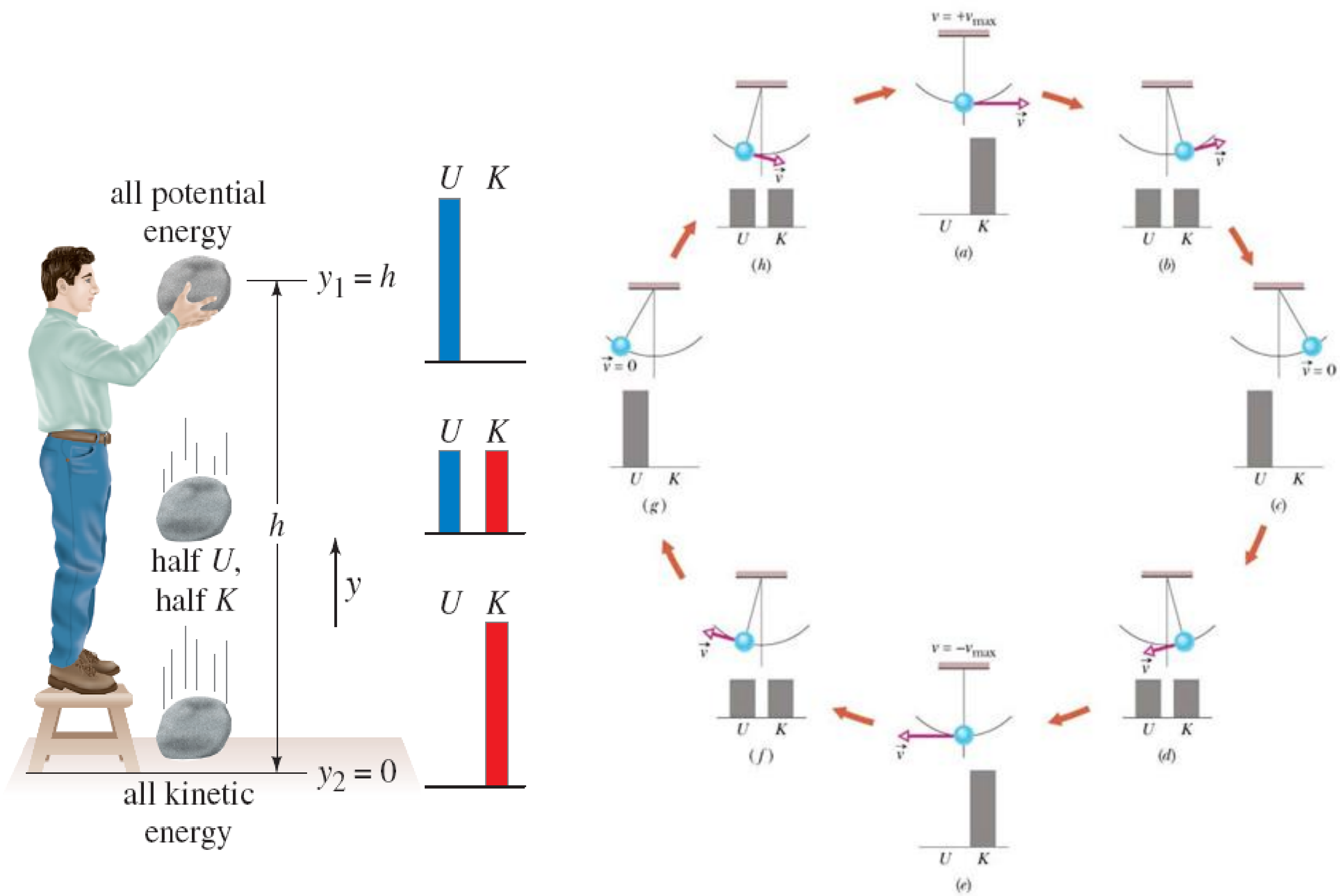
By rearrangement of last equation, we can arrive with

$$KE_2 + PE_2 = KE_1 + PE_1,\ or$$
$$E_2 = E_1 \tag{3.7}$$

Equation 3.8 defines the **law of conservation of mechanical energy**: energy can neither be created nor destroyed, but can be transforming from one form (PE) to another(KE) and vice versa.

Example 3.6 (energy transformation and conservation of energy**)**

Consider the examples of energy transformation and conservation of energy shown in Figure 3.4.



(a) (b)

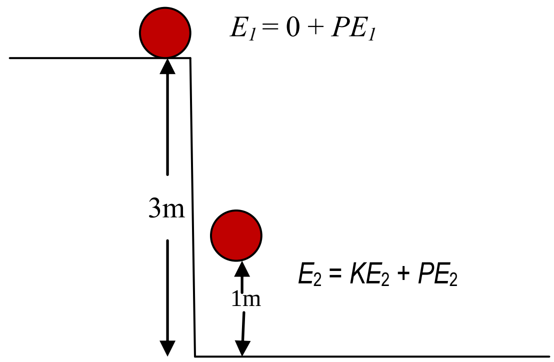
Figure 3.4: (a) a man dropped a stone (free fall object) from height h above ground (y_2=h=0). (b) a pendulum swinging without air resistance and friction.

Both cases (a) and (b) of Figure 3.4 illustrates energy transformation from KE to PE and vice versa. At any moment, energy is conserved.

Example 3.7

A 1kg ball is lifted to a height of 3m above the ground. Calculate (a) the ball's KE when it has fallen to a 1 m above the ground and (b) the ball's kinetic energy at the moment it hits the ground. Assume no air resistance

Solution



Given Data

$$m = 1kg, d_1 = 3m, d_2 = 1m, \; KE\,?$$

No need for FBD

Because there is no air resistance, then we can apply the law of conservation of energy. Consider two points of interest: point 1 at 3m with energy E_1 and point 2 at 1m above the ground at E_2.

(a)

$$\because E_1 = E_2$$
$$\therefore \; KE_1 + PE_1 = KE_2 + PE_2$$
$$0 + PE_1 = KE_2 + PE_2$$
$$KE_2 = PE_1 - PE_2 = mgd_1 - mgd_2 = mg(d_1 - d_2) = (1kg)(9.8m/s^2)(3-1) = 19.6J$$

(b)

When the ball hits the ground all the PE_1 at point 1 has been transformed into KE, which will be converted to sound and heat.

3.5 Simple Machines

A simple machine is a mechanical device that multiplies an applied force. In any simple machine, there are two forces involved

- The applied force to the machine or the **effort**
- The force that the machine is to overcome or the **resistance**

In general the effort force is always smaller than the resistance force.

3.5.1Examples of Simple Machines

Two most important simple machines

- **Lever**
- **Pulley**

3.5.1.1The lever

shown in Figure 3.5 consists of a rigid bar or stick free to turn on a pivot called fulcrum. It has two ends and is used to lift a heavy objects. A small force $\mathbf{F_1}$ **(**$\mathbf{F_{effort}}$**)** at one end generates a large force $\mathbf{F_2}$ **(**$\mathbf{F_{load}}$**)** to lift the object. The distances d_1 and d_2 represent the effort and resistance distance from the pivot.

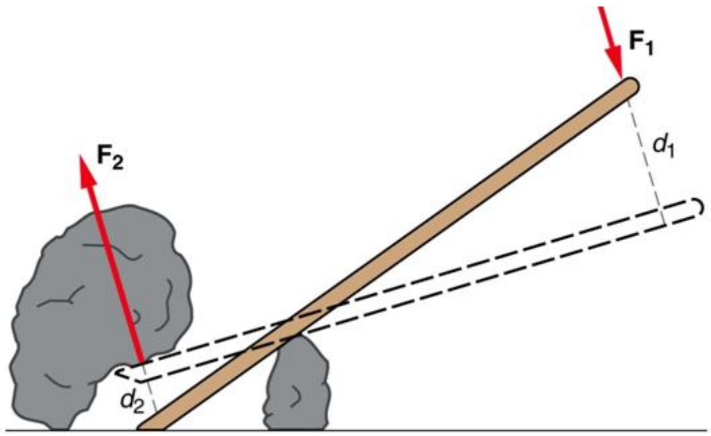


Figure 3.5: The lever, a simple machine to lift a heavy object by exerting a small force.

Simple Machine Law

By ignoring friction, the simple machine works according to a principle or law, which can be stated as

effort force × effort distance = resistance force × resistance distance, or

$$F_e \times d_e = F_r \times d_r \qquad (3.8)$$

This implies that

- F_e is always less than F_r and d_e is always larger than d_r
- work input equals the work output

Mechanical Advantage (MA**)**

The **mechanical advantage** MA is the ratio of resistance force to the effort force, or effort distance to resistance distance. That is

$$MA = \frac{F_r}{F_e} = \frac{d_e}{d_r} \qquad (3.9)$$

Notice that *MA* is a unitless (has no unit) and always greater than 1

Example 3.8

A person applies 30 N on the car jack (a lever) and the machine produce 600 N lifting force on the car. What is the jack's mechanical advantage?

Solution

Look at Figure 3.5, the stone is now a car

Given Data

$F_e = 30N, F_r = 600N,\ MA?$

$$\because MA = \frac{F_r}{F_e}$$

$$\therefore MA = \frac{600N}{30N} = 20$$

A mechanical advantage of 20 means that the machine (jack) multiplies the input force of 30 N by 20 times and makes it 600 N (Whoa)

3.5.1.2 The pulley

A pulley is a grooved wheel that turns on an axle and is supported on a frame. It can be suspended to a fixed point(ceiling, for example) or movable object (load or resistance) as shown in Figure 3.6.

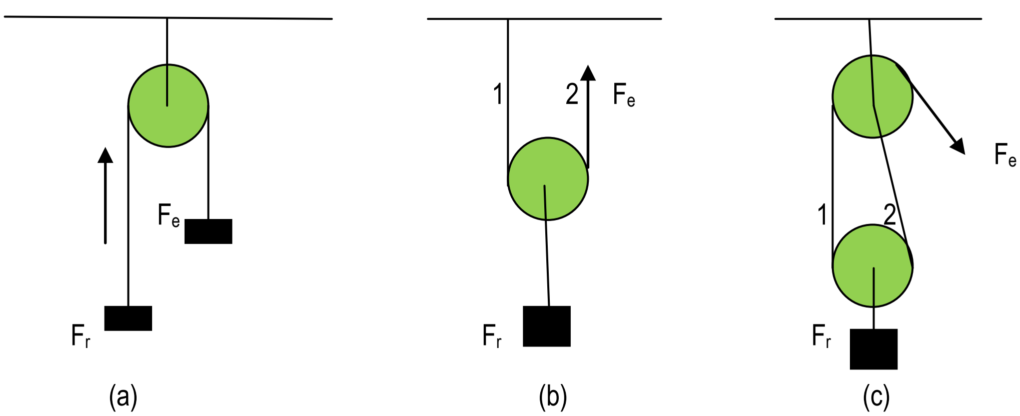


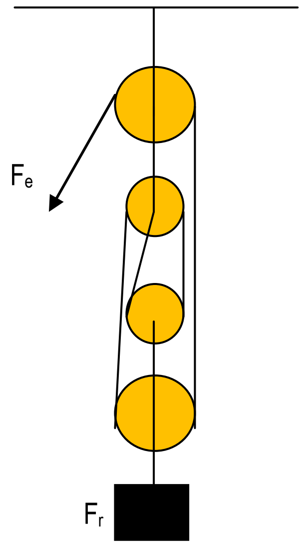
Figure 3.6 (a) one fixed pulley, (b) one movable pulley, and (c) a combination of one fixed and one movable pulley system.

The law of simple machine as well as the mechanical advantage apply also to pulleys. The purpose of using a fixed one pulley is to change direction and thus has no mechanical advantage or MA = 1. A flag pole is an example of fixed pulley . The pulley (b) and the pulley system (c) of Figure 3.6 both have mechanical advantage of 2. A simple and most convenient way to find MA is counting the number of strings that support the movable pulley or system of movable pulleys. The numbers 1 and 2 indicate the number of strings.

Example 3.10

Determine the mechanical advantage of system of pulleys shown in the drawing below. If F_r is 1000 N how much then is F_e

Solution: MA = 4 (prove it) and Fe = 250 N (prove it)



SUMMARY OF CHPTER 3

work is force times distance or the mechanical transfer of energy to or from an object. There are always two objects involved, a giver and loser of energy. A positive work is energy gained by object and a negative work is energy lost or given off by an object to another object. Force is the tool of transfer energy. The applied force must be parallel to the distance traveled for the energy to be transferred, Forces like gravity and the normal force, which are perpendicular to the horizontal motion/distance traveled do not do work. *Energy* is the ability to do work. Work and energy are related and share the same unit of measurement, the Joule. Mechanical energy consists of two parts: kinetic and potential energy. *Kinetic energy* (*KE*) is the energy of motion and *potential energy (PE)*is the energy of position. *KE* can be transformed into *PE*, and *PE can be transformed into KE*. In a conservative system, a one under a conservative force only, the sum of both *KE* and *PE* of remains unchanged during a process; the sum at the beginning of the process equals the sum at the end of the process. This is the law of *conservation of energy*. *Power* is how quick work is done and it is measured in units of watt or most commonly in kilowatt (kW).

Simple machine is a mechanical device that multiplies input forces. Levers and pulleys are just two common examples of simple machines. By simply applying a small force at one end of the lever, a larger force can be exerted on the load or resistance attached to the other end. The only price one should pay by using a lever or a pulley is that the effort distance must be larger than the resistance distance. The pulley achieves the same purpose as of lever. The law of simple machine is a manifestation of mechanical equilibrium. The energy (work) on both sides of the equation are equal.

Basic Equations

Work is force times distance: $W = F \times d$ (3.1)

Gravitational Potential Energy is energy of position: $PE = mgd$ (3.2)

Kinetic energy is the energy of motion: $KE = \frac{1}{2}mv^2$ (3.3)

Power is work per unit time: $P = \frac{W}{t}$ (3.4)

Conservation of energy:

$$KE_2 + PE_2 = KE_1 + PE_1, \ or$$
$$E_2 = E_1 \quad (3.7)$$

mechanical advantage: $MA = \frac{F_r}{F_e} = \frac{d_e}{d_r}$ (3.9)

Basic Principles

Law of Conservation of Mechanical Energy: If there is no friction or resistance to motion, then the mechanical energy $E = KE + PE$ of an object is constant during any process. That is

$$KE_1 + PE_1 = KE_2 + PE_2$$

Law of simple machines work input = work output: $F_e \times d_e = F_r \times d_r$

Chapter 3 Worksheet

Part1: Sentence Completion

1. Work is ------------------ times ------------------.
2. Energy is the ---.
3. Potential energy is the ---------------- of -------------------.
4. Kinetic energy is the -------------------- of ----------------------.
5. Power is --.
6. Mechanical advantage is ----------------------------- divided by ---------------------------------

Part 2: Multiple Choices

1. Work is done when

   A. A force is applied.
   B. A person tries unsuccessfully to move a box.
   C. Force is applied and an object moved.
   D. None of the above is correct

2. Kinetic energy is

   A. Energy of motion.
   B. Energy of position.
   C. Energy stored in fossil fuels.
   D. Electrical energy.

3. When you through a ball straight up in the air, its KE

   A. Is $1/2mv^2$.
   B. equals the work you did on the ball when you through it.
   C. Is converted into gravitational potential energy as it goes higher.
   D. All of the above is correct.

4. A ball falls off a window 10 m above the ground. Its KE as it hits the ground is

   A. The same as its GPE it had before falling.
   B. Equal to the work done in placing the ball on the window.
   C. Equal to the loss of GPE as a result of the fall.

D. All of the above is correct.

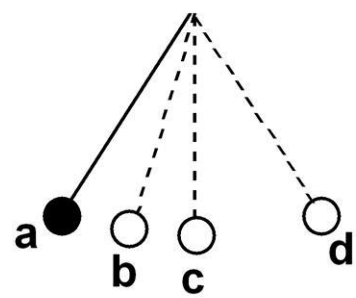
5. The important variables in the GPE equation are the weight of an object and

A. Its distance from the ground.
B. Its speed.
C. Its mass.
D. All of the above is correct.

6. The rate at which work is done is

A. Acceleration.
B. Potential energy.
C. Kinetic energy.
D. Power.

7. The pendulum shown in the drawing is being pulled up to position, then released. Where is its KE maximum?



A. At a.
B. At b.
C. At c.
D. At d.

8. When you stop at the top after climbing a flight of stairs, you have

A. Lost energy.
B. Lost work.
C. Gained PE.
D. Gained KE.

9. The KE of a moving object can be defined as

A. How much work was done putting the object into motion.
B. How much work is needed to bring the object to a stop.
C. $\frac{1}{2} mv^2$.
D. All of the above is correct.

10. Which of the following occurs when one rides a bicycle?

A. The bicycle has KE because of its motion.
B. The KE is obtained from the conversion of chemical energy stored in the rider’s muscles.

C. The energy for riding the bicycle was originally generated in nuclear reactions in the sun.
D. All of the above is correct.

11. Which of the following is not a simple machine?

A. Lever.
B. Pulley.
C. Car.
D. System of pulleys.

12. The force applied to a simple machine is the

A. Effort.
B. Resistance.
C. Friction.
D. Normal.

13. A pulley system has eight strands holding the load or resistance. The mechanical advantage is

A. 4
B. 8
C. 16
D. 64

Part3: True/False

1. Work is the rate of doing energy.

A. True
B. False

2. The energy an object has because of its position is called GPE.

A. True
B. False

3. You do more work on yourself when you run up the stairs than when you walk slowly.

A. True
B. False

4. Energy is not conserved when an object slows to a stop.

A. True
B. False

5. The watt is a unit of work.

A. True
B. False

6. The unit of power is Joule.

   A. True
   B. False

7. A car at rest on ground possesses a large GPE.

   A. True
   B. False

8. When there is no air resistance and no friction, a pendulum would swing forever.

   A. True
   B. False

9. Work is a vector quantity.

   A. True
   B. False

10. Work can be done by a moving object by itself.

   A. True
   B. False

11. The resistance is the name given to the force overcome by the simple machine.

   A. True
   B. False

12. The mechanical advantage of a pulley depends on the diameter of the pulley.

   A. True
   B. False

Part4: Questions

1. Equal forces acting on boxes A and B and move them across the floor. Box A has twice the mass of box B, but box B moves twice the distance moved by box A. Which box has the greater amount of work done on it? Explain.

2. A person pushes very hard for several seconds on a heavy table, but the table does not move. Has the person done any work on the table? Explain

3. Can KE energy be negative? Explain

4. When a rubber ball is dropped from 1m above the ground, can it rebound to a height grater than its original 1m height? Explain

5. Where does the KE come from when a car accelerates uniformly from rest? How is the increase in KE is related to the friction force the road exerts on the tires?

6. At what point is the GPE of a swinging pendulum ball at a maximum?

7. Can an object possess both KE and PE at the same time? Explain

8. Why is a person likely to be severely injured by a bolt falling from the fourth floor of a construction site than one falling the second floor? Explain

9. What is the difference between a fixed pulley and a movable pulley?

10. State the law of simple machines.

11. State the law of mechanical advantage.

Part5: Exercises

1. How high can a 10 kg mass be lifted by a 1000 J of work?

2. A 20 kg box is carried up a 2500 m high mountain in 5 h. (a) how much work is done and (b) how much is power in kilowatt.

3. At what speed does a 10 kg mass have a KE of 100 J?

4. The GPE of a 50 kg mass after being lifted to the top of a building is 500 J. How high is the building?

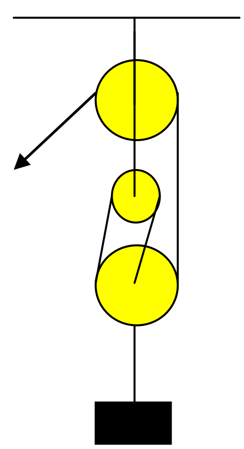
5. A man uses a lever to lift a box. The box has a resistance of 300N while the man exerts an effort force of 100 N. What is the mechanical advantage of the lever?

6. A pulley system has a mechanical advantage of 6. What is the resistance force if an effort of 135 N is applied?

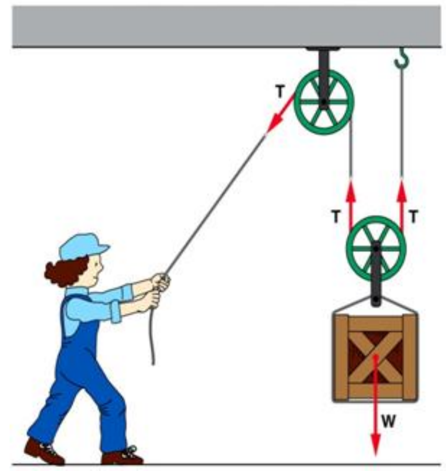
Part 6: Challenging Exercises

1. A system of two fixed pulleys and two movable pulleys ha a mechanical advantage of 4. (a) If a force of 88 N is applied, what weight (load or resistance) is raised? (b) If the weight is raised 10.5 m, what length of rope is pulled?

2. Find the mechanical advantage of the following pulley system



3. What is the mechanical advantage of this system of pulleys?



4. (a)How much work is done in lifting the 200 N block of ice shown below a vertical distance of 2.5 m? (b) how much work is done in pushing the same block of ice up the 5 m long ramp? The force needed is only 100 N, and (c) what is the increase in the block's gravitational potential energy in each case?

Fcredit: P.G. Hewitt, J.Suchocki, and L.A. Hewitt, " *Conceptual Physical Science-Explorations*".