# CHAPTER 4

# IMPULSE AND MOMENTUM

## CHAPTER'S OBJECTIVES

- To understand the interaction between objects through the impulse and momentum concepts
- To introduce the law of conservation of momentum, and apply it to solve collision problems
- To understand the two aspects of collision, the *elastic* and *inelastic*

In Chapter 1 we studied motion from experimental point of view . In Chapter 2 we associate motion with force and developed a new definition of force a s an *interaction* between objects in contact or non contact. definition of force. The mechanism of this interaction in terms of *work* and *energy* was the focus in chapter 3 .

In light of this, we now examine the *collision* process between two interacting objects using new concepts of impulse and momentum.

**4.1 Impulse** $J$ **:** Collision Problem

**Collision** is defined as a short time interaction between two objects in contact, such as a tennis ball and a racket. Below is a brief description of this collision

- When the ball (object A) hits the racket (object B), they stay in contact for a short period of time $\Delta t = t_f - t_i$ . Typically, $\Delta t$ is between 1 to 10ms.
- The forces $\vec{F}_{A\,on\,B} = \vec{F}_{B\,on\,A}$ are action and reaction pair
- The force is not constant, but varies with time. We use the term average force to identify this collision force

**Impulse** $J$ is the product of the average force and the time of contact and can be written as

$$\vec{J} = \vec{F}\Delta t \tag{4.1}$$

Equation 4.1 implies the following

- When the tennis ball got hit by the racket it receives an impulse
- Impulse is a vector quantity points always in the direction of the force
- The stronger the force acting, and the longer the time of contact, the greater the impulse will be

The metric (SI) unit of impulse is Newton times second (N.s).

**Example 4.1**

An average force of 300 N acts for a time of 0.05 s on a golf ball. What is the magnitude of the impulse acting on the ball?

Solution

Given data:

$F = 300N$

$\Delta t = 0.05s, \quad J\,?$

$\because J = F\Delta t$

$\therefore\ J = (300N)\cdot(0.05s) = 15\,N.s$

**4.2 Momentum** $p$

The product of mass and velocity is called **momentum** $p$ . It can be written as

$$\vec{p} = m\vec{v} \tag{4.2}$$

Equation (4.2) implies the following

- An object at rest has zero momentum
- The faster the object is moving or the more mass it has, the greater momentum it has.
- Momentum a vector quantity points in the direction of velocity. If the direction is not important, then we can drop the vector notation on momentum and speak of speed instead of velocity.

The metric (SI) unit of momentum is a unit of mass times a unit of velocity (kg.m/s).

**Example 4.2**

Find the momentum of 1500 kg car traveling at 10m/s.

Solution

Given data:

$$mass = 1500kg, \quad v = 10m/s$$
$$wanted: p?$$

$$\because p = mv$$
$$\therefore \; p = (1500kg)\cdot(10m/s) = 15000kgm/s = 1.5\times10^4\,kgm/s$$

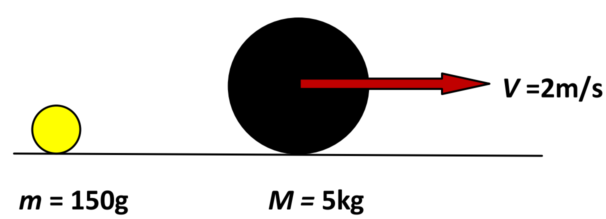
**The Relationship between Momentum and Inertia (mass)**

Momentum is related to both mass and speed. Therefore, it is much easier to stop a small object (bicycle, for example) than a truck both traveling at same speed. The truck, in this respect, has more inertia and is more difficult to stop or to move than the small object.

**Example 4.3**

The drawing below shows a bowling ball of mass 5kg travels at 2m/s and a tennis ball with mass of 150g. Can both balls have the same momentum? If yes at what speed must the tennis ball travel to have the same momentum?

Solution




Given data:

$$m = 150g = 0.150kg, \; M = 5kg, \; V = 2m/s$$
$$p_{BB} = p_{TB}, \; v$$
$$Wanted: v(tennis\,ball)$$

$\because MV = mv$

$\therefore v = \frac{MV}{m} = \frac{(5kg)(2m/s)}{0.150kg} = 66m/s$

Therefore, the tennis ball must be moving with speed 66m/s to have same momentum as the bowling ball.

**4.3 The Impulse-Momentum Theorem**

The impulse-momentum theorem states that: impulse changes momentum, or

$$J = \Delta p \qquad (4.3)$$

Equation (4.3) can be rewritten as

$$F\Delta t = p_f - p_i = mv_f - mv_i \qquad (4.4)$$

Equation (4.4) implies the following

- $\Delta$ means a change $\Rightarrow \Delta = final - initial$
- To increase momentum, i.e., $p_f > p_i$, increase $F$, or $\Delta t$, or both
- To decrease momentum, i.e., $p_f < p_i$, increase or decrease $\Delta t$
  - ✓ Increasing $\Delta t \Rightarrow$ less force $F$
  - ✓ Decreasing $\Delta t \Rightarrow$ large force $F$

Decreasing and increasing the time of contact has a lot of applications. For example, reducing the time yield a large force that breaks a stack of bricks shown in Figure 4.1. The auto air bag is designed to protect the driver from severe head damage during collision. The idea behind the bag is to increase the time of collision and therefore reduce the force of collision to a minimum.

(a)

(b)

Figure 4.1: (a) In a short time of impact, a large force is produced to break a target. (b) In a long time of impact, a small force is produced, which reduces head injuries of the driver and passenger.

**Example 4.4**: A tennis ball bouncing off the floor.

Examine the impulse-momentum change theorem by considering a tennis ball bouncing off the floor as shown in Figure 4.2

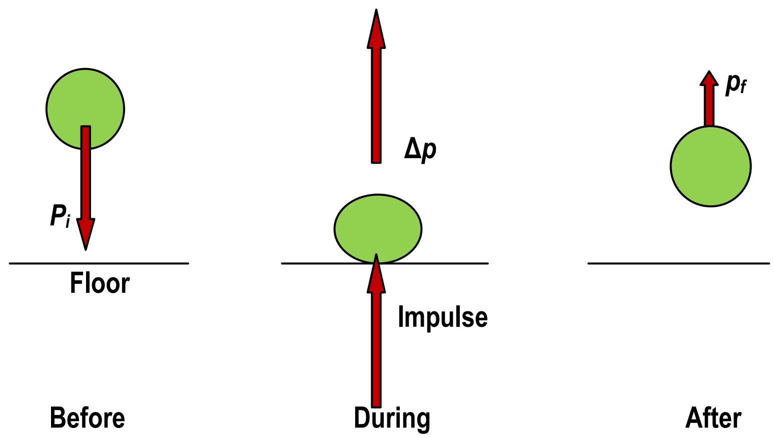



**Figure 4.2** A typical collision problem of a tennis ball bouncing off a floor.

- When the ball hits the floor, the momentum decreases to zero as the ball comes to a temporary stop
- The ball receives an impulse $J$ from the floor in upward direction
- The ball gains momentum in opposite direction (upward)
- The ball leaves with $\Delta p = p_f - p_i = p_f - 0 = p_f$

**Example 4.5**

A baseball of mass 0.15 kg has an initial velocity 0f 20 m/s as it approaches a bat as shown in Figure 4.3. It is hit straight back to the right and leaves the bat with a final velocity of +40 m/s. (a) Determine the impulse applied to the ball by the bat. (b) Assume that the time of contact is $1.6\times10^{-3}$ sec , find the average force exerted on the ball by the bat. (c) How much is the impulse exerted by the ball on the bat?

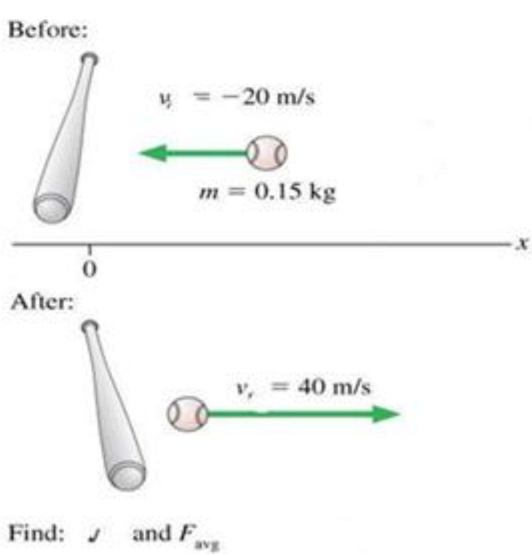



**Figure 4.3**: A collision between a ball and bat. Credit: *D. Knight* "*Physics for scientists and engineers a strategic approach*" by Pearson-Addison Wesley 2004.

(a) Apply the impulse-momentum theorem

$$J = mv_f - mv_i = (0.15kg)(40m/s) - (0.15kg)(-20m/s) = 9\,kg\,m/s$$

(b) Apply the equation that defines impulse

$$\because J = F\Delta t$$

$$\therefore F = \frac{J}{\Delta t} = \frac{9\,kg\,m/s}{0.0060s} = 1500N$$

(c) Apply Newton's third law of action and reaction and get

Impulse exerted on the bat by the ball equals -9 kg m/s. The negative sign indicates a direction to the left of the origin of coordinate system.

**Example 4.6**

When you jump from a height (say 3m) what kind of adjustment should you do to reduce sever injuries?

Solution

Bend your knees when landing. How you analyze the situation?

Recall the equation

$$F\Delta t = \Delta p,\ rearrange and\ get$$

$$F = \frac{\Delta p}{\Delta t}$$

Bending knees (say 5 cm) increases the time of impact with the ground and therefore decreases the force *F* on the feet and legs. Landing on the ground with stiff-legged (say with 1 cm body move) decreases the time and increasing *F*. Typical exercise will show that the ground force on the leg of a 70 kg person is around $2.1\times10^5$ N , while in bending the knee the force on the same person is about $4.2\times10^3$N.

Clearly, the force on the feet and legs is much less when the knees are bent. Indeed, the strength of the leg bone is not great enough to support the force given above, so the leg would most likely break in such a stiff landing.

**4.4 Law of Conservation of Momentum**

The law of conservation of momentum is a direct result of Newton's third law. It applies when there are no net external forces acting on the objects involved. To show this, let us take an example of two balls collided a head-on collision as is shown in Figure 4.4. The only forces acting are internal forces.

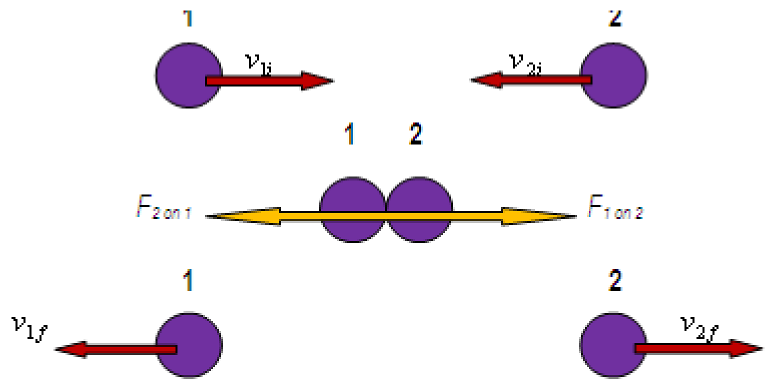



**Figure 4.4** A collision between two objects.

By using the impulse=momentum theorem, we can write the forces acting on both balls as follows

$$F_{1\,on\,2} = \frac{\Delta p_1}{\Delta t},\ and$$
$$F_{2\,on\,1} = \frac{\Delta p_2}{\Delta t} = -F_{1\,on\,2}$$

By adding these two equations we get

$$F_{1\,on\,2} + F_{2\,on\,1} = \frac{\Delta p_1}{\Delta t} + \frac{\Delta p_2}{\Delta t} = 0,\ \ or$$
$$\frac{\Delta}{\Delta t}(p_1 + p_2) = 0$$

If the time change of the quantity $(p_1 + p_2) = 0$, then this quantity is constant and doesn't change with time, or

$$p_1 + p_2 = constant \qquad (4.5)$$

Equation (4.45)is a statement of a **conservation law of momentum**: if $p_1 + p_2 =$ constant, then the sum of the momentum before collision equals to the sum of momentum after the collision. That is,

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \qquad (4.6)$$

**Example 4.7**

A 10 g bullet is fired from a 3 kg rifle with speed of 500 m/s, as shown in Figure 4.5. What is (a) the initial momentum of the system (bullet and rifle)? And (b) the recoil speed of the rifle?

Solution

Given Data**:**

$m_b = 0.010kg,\ m_r = 3kg$

$(v_b)_f = 500m/s,\ \ wanted: (v_r)_f\ ?$


**Figure 4.5:** The rifle and bullet**.** Credit: *B.W. Tillery, E. D. Enger, and F. C. Ross, "Integrated science", 3rd Ed., McGraw Hill 2004.*

$$(a)$$
$$p_b)_i = m_b \times v_{bi} = (0.010g) \times 0 = 0$$
$$p_r)_i = m_r \times v_{ri} = (3kg) \times 0 = 0$$
$$\therefore\ p_{bi} + p_{ri} = 0 + 0 = 0$$

The law of conservation of momentum states that the initial momentum of the system equals the final momentum, or

$$p_{bf} + p_{rf} = p_{bi} + p_{ri} = 0,$$
$$\therefore\ p_{bf} + p_{rf} = 0,\ \ or$$
$$p_{rf} = -p_{bf} = -(0.010kg) \times (500m/s) = -5kg \cdot m/s$$

The negative sign indicates that the rifle's recoil is to the left with recoil velocity

$$\because\ p_{rf} = m_r \times v_{rf}\,,\quad \therefore\ v_{rf} = \frac{-5kgm/s}{3kg} = -1.67m/s$$

### 4.5 Collisions

The collision of two objects involves very large internal forces acting for very short period of time. To comprehend this topic, two different kinds of collision will be discussed here.

#### 4.5.1 Elastic Collision

- Is a collision in which the total kinetic energy of the collided objects after collision equals the total kinetic energy before collision

- The collided object bounce a part and return to their original shape without a permanent deformation

### 4.5.2 Inelastic Collision

- Is a collision in which the total kinetic energy of the collided objects after collision is not equal to the total kinetic energy before collision. The two object experience a permanent deformation in their original shape

- In completely inelastic collision, the two objects coupled and move as a one object after collision

  The kinetic energy is lost in two ways

**1.** It can be converted into heat because of friction.

**2.** It is spent in doing permanent deformation as in cars and part of it goes also as sound.

**In both elastic and inelastic collisions, the total momentum of both objects is conserved.**

**Example 4.8** elastic collision

Figure A ball of mass 0.6 kg traveling at 9 m/s to the right collides head on collision with a second ball of mass 0.3 kg traveling at 8 m/s to the left. After the collision, the heavier ball is traveling at 2.33 m/s to the left. What is the velocity of the lighter ball after the collision?


**Figure 4.6**: an elastic collision between two moving objects.

Solution

Given Data

$$m_1 = 0.6kg, v_1 = +9m/s,\ m_2 = 0.3kg, v_2 = -8m/s, v_1' = -2.3m/s,$$
$$wanted:\ v_2'\ ?$$

$$\because m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$\therefore\ v_2' = \frac{m_1 v_1 + m_2 v_2 - m_1 v_1'}{m_2} = \frac{(0.6kg)(9m/s) + (0,3kg)(-8m/s) - (0.6kg)(-2.3m/s)}{0.3kg}$$
$$\therefore\ v_2' = +14,6m/s$$

**Example 4.9** completely inelastic collision

Figure 4.7 shows a $1.75\times10^4$ kg railroad car traveling at 8 m/s to the east as shown in the drawing below is collided with another car of the same mass and initially at rest and couple with it. What is the velocity of the coupled system of cars after the collision?

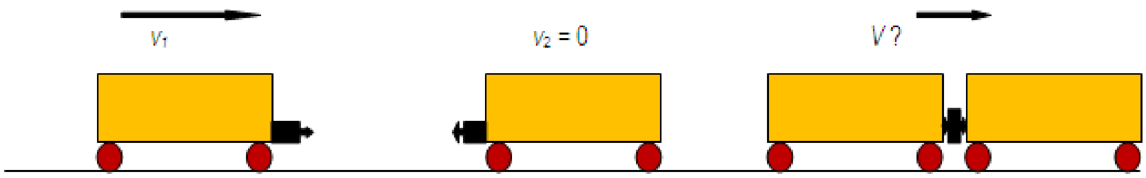



**Figure 4.7**: an inelastic collision between two objects. After the collision the two objects coupled and move as one object with speed $V$ .

Solution

Given Data

$$m_1 = m_2 = 1.75\times10^4 kg,\ v_1 = 8m/s,\ v_2 = 0,$$
$$wanted:\ \ V?$$

$$\because m_1v_1 + m_2v_2 = (m_1 + m_2)V$$
$$\therefore V = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{m_1v_1 + 0}{m_1 + m_2} = 3.5m/s$$

# SUMARRY OF CHPTER 4

*Collision* is a short time interaction between two objects. It involves very large and time dependent internal forces that dominate all external forces. Being action and reaction forces, these forces transfer energy from one object to another. *Impulse J* is the product of the interaction force and the time of collision. It is a vector, which points always in the direction of the force. *Momentum* is mass times velocity and is a measure of the object's motion. It is a vector, which always points in the direction of velocity. Momentum can be increased by either increasing the object's mass, or its speed ,or both mass and speed. The *impulse-momentum theorem* states that the impulse equals the change in momentum. If the external forces acting on the two interacting objects are ignored, then the total momentum before interaction equals the total momentum after collision. This is the *law of conservation of momentum; momentum before equals momentum after*. Collision problems are classified as either elastic collision or inelastic collision. In *elastic collision* both momentum and kinetic energy are conserved. In *inelastic collision* momentum only is conserved, where the object loses energy in the form of heat and sound.

**Basic Equations**

Impulse is force times time: $\vec{J} = \vec{F}_{avg} \times \Delta t, \ or \ \ J = F\Delta t$ (4.1)

Momentum is mass times velocity: $\vec{P} = m\vec{v}, \ or \ P = mv$ (4.2)

**Basic Principles**

Impulse –momentum theorem impulse equals change of momentum:

$$J = \Delta p, \ or \ F\Delta t = P_f - P_i \quad (4.4)$$

Conservation of momentum: momentum before equals momentum after:

$$P_i = P_f\,, \ or \ m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \quad (4.6)$$

# Chapter 4 Worksheet

**Part1: sentence completion**

1. Impulse is ____________ times ______________.
2. Momentum is ____________ times_____________.
3. More inertia implies more_____________.
4. _____________ equals change of momentum.
5. Energy ___________ equals energy ____________. This is the law of ________________________.
6. Momentum_____________ equals momentum____________. This the law of__________________.

**Part2: Multiple choices**

1. Impulse is

   **A.** A force applied to an object.
   **B.** The initial force applied to an object.
   **C.** The initial momentum applied to an object
   **D.** The change in momentum due to a force being applied to an object during a short period of time.

2. Momentum is

   **A.** Equal to speed times weight.
   **B.** Equal to mass times velocity.
   **C.** Another alternative name to force
   **D.** All of the above is correct.

3. What is the metric (SI) unit of momentum?

   **A.** Kg m/s.
   **B.** Newton.
   **C.** Kg $m/s^2$
   **D.** Newton. meter

4. Which of the following statement is correctly expressing the conservation of total momentum of interacting objects?

   **A.** The total momentum always remains the same.
   **B.** The total momentum remains the same if there are no internal forces.
   **C.** The total momentum remains the same if there are no external forces.
   **D.** No one of the above is correct.

**5.** A 2000 kg car is moving to the right at 30m/sec and collided with a wall and comes to rest at 0.2 sec. The average force the car exerts on the wall is

**A.** $1.2 \times 10^4$ N to the right
**B.** $3 \times 10^5$ N to the left
**C.** $6 \times 10^4$ N to the right
**D.** None of the above

**6.** A 3kg object moves to the right with a speed of 4 m/sec. It collides in a perfectly elastic collision with a 6kg object moving to the left at 2m/sec. What is the total kinetic energy after the collision?

**A.** 72 J
**B.** 36 J
**C.** 24 J
**D.** 0 J

**7.** How long must a 100 N force act to produce a change in momentum 200 kg.m/sec?

**A.** 0.25 sec
**B.** 0.50 sec
**C.** 1.0 sec
**D.** 2.0 sec

**8.** Which is a vector quantity?

**A.** Energy
**B.** Work
**C.** Power
**D.** Momentum

**9.** When the velocity of an object is doubled, its ____________ is also doubled

**A.** Gravitational potential energy
**B.** Acceleration
**C.** Momentum
**D.** Kinetic energy

**10.** Impulse is related to

**A.** Kinetic energy
**B.** Change in kinetic energy
**C.** Momentum
**D.** Change in momentum

**Part 3: True of false**

**1.** The impulse is always in the same directions as the average force.

**A.** True
**B.** False

2. The momentum of an object remains the same when the net external force acting on it is zero.

   A. True
   B. False

3. If the kinetic energy of an object is zero, then its momentum must not be zero.

   A. True
   B. False

4. A large force always produces a larger impulse on a body than a smaller force.

   A. True
   B. False

5. The kinetic energy is always conserved both in elastic collisions and inelastic collisions.

   A. True
   B. False

6. If two particles of different masses have equal kinetic energy they also have equal momentum.

   A. True
   B. False

**Part3: Exercises**

1. An average force of 250 N acts on a ball for 0.05 sec. (a) What is the magnitude of the impulse on the ball? (b) What is the change in the ball's momentum?

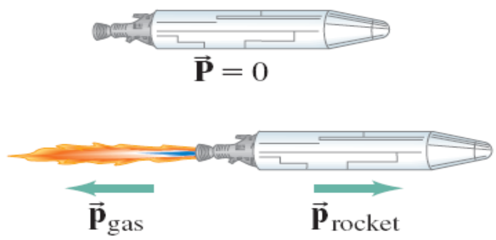
2. What is the momentum of 2000 kg truck traveling with 25m/sec?

3. A football of mass 1.5 kg and a speed of 2 m/sec and ping pong ball of mass 0.003kg and a speed of 4 m/sec. Which ball has the larger momentum?

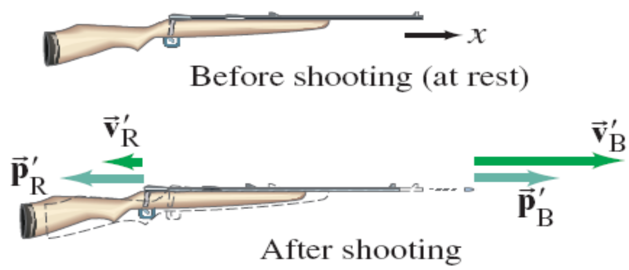
4. A force acts on a ball initially at rest for 0.05 sec. (a) What is the impulse on the ball? (b) What is the final momentum of the ball?

5. A father ice skater with a mass of 80kg pushes off against his child ice skater whose mass is 45kg. Both skaters were initially at rest. (a) What is the total momentum of both skaters after they push off? (b) If the father skater moves off with speed of 1.5 m/sec, what is the speed of the child?

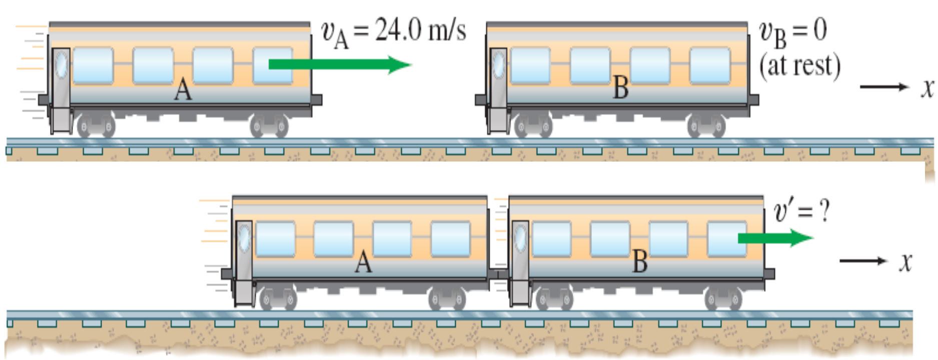
6. A rocket ship, shown in the drawing below, at rest in space gives a short blast of its engine, firing 50kg of exhaust gas out the back end with an average velocity of 200 m/sec. What is the change in momentum of the rocket during this blast?




7. A rifle and a bullet shown in the drawing below. The rifle of  mass of the rifle 1.5 kg fires a bullet of 5g mass. The bullet moves with muzzle velocity **v**′= 500m/sec. (a) What is the momentum of the fired bullet? (b) If the external forces acting on the rifle are ignored, what is its recoil velocity after firing the bullet?




8. Consider the collision of train cars shown below as a perfectly inelastic collision. find the velocity of the coupled cars after collision. Assume a 10,000 kg mass of each car




**Part4: Challenge exercises**

1. A bullet is fired into a block of wood sitting on a block of ice. The bullet has an initial velocity of 350m/sec and a mass of 50g. The wooden block has a mass of 2 kg and is initially ate rest. The bullet remains inside the wooden block after collision. (a) assuming the momentum is conserved,

find the velocity of wood and bullet after the collision. (b) What is the magnitude of the impulse that acts on the block of wood?

**2.** A 2000 kg ca is traveling north with speed of 30 m/sec collides head on with a 4000kg truck traveling south with speed of 20 m/sec. The car and truck stick together after the collision.

(a) What is the total momentum of the system of car and truck before collision?
(b) What is the velocity of both just after the collision?
(c) What is the total kinetic energies of the system before collision?
(d) What is the total kinetic energies just after the collision?
(e) Is the collision elastic or inelastic? Explain