Chapter 10

$s = r\theta$	$v = r\omega$
$1 \text{ rad} = 360°/2\pi \approx 57.3°$	$a_t = \frac{dv}{dt} = r\frac{d\omega}{dt}$
$\overline{\omega} \equiv \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$	$a_t = r\alpha$
$\overline{\alpha} \equiv \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$	$a_c = \frac{v^2}{r} = r\omega^2$
$\omega_f = \omega_i + \alpha t \quad \text{(for constant } \alpha)$	$I \equiv \sum_i m_i r_i^2$
$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 \quad \text{(for constant } \alpha)$	$K_R = \frac{1}{2} I\omega^2$
$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i) \quad \text{(for constant } \alpha)$	$\tau \equiv rF\sin\phi = Fd$
$\theta_f = \theta_i + \frac{1}{2}(\omega_i + \omega_f)t \quad \text{(for constant } \alpha)$	

1. During a certain period of time, the angular position of a swinging door is described by $\theta = 5.00 + 10.0t + 2.00t^2$, where θ is in radians and t is in seconds. Determine the angular position, angular speed, and angular acceleration of the door (a) at $t = 0$ (b) at $t = 3.00$ s.

(a)

$$\theta\big|_{t=0} = \boxed{5.00 \text{ rad}}$$

$$\omega\big|_{t=0} = \frac{d\theta}{dt}\bigg|_{t=0} = 10.0 + 4.00t\big|_{t=0} = \boxed{10.0 \text{ rad/s}}$$

$$\alpha_{t=0} = \frac{d\omega}{dt}\bigg|_{t=0} = \boxed{4.00 \text{ rad/s}^2}$$

(b)

$$\theta\big|_{t=3.00 \text{ s}} = 5.00 + 30.0 + 18.0 = \boxed{53.0 \text{ rad}}$$

$$\omega\big|_{t=3.00 \text{ s}} = \frac{d\theta}{dt}\bigg|_{t=3.00 \text{ s}} = 10.0 + 4.00t\big|_{t=3.00 \text{ s}} = \boxed{22.0 \text{ rad/s}}$$

$$\alpha\big|_{t=3.00 \text{ s}} = \frac{d\omega}{dt}\bigg|_{t=3.00 \text{ s}} = \boxed{4.00 \text{ rad/s}^2}$$

2. A dentist's drill starts from rest. After 3.20 s of constant angular acceleration, it turns at a rate of 2.51×10^4 rev/min. (a) Find the drill's angular acceleration. (b) Determine the angle (in radians) through which the drill rotates during this period.

$$\omega_f = 2.51 \times 10^4 \text{ rev/min} = 2.63 \times 10^3 \text{ rad/s}$$

(a)

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{2.63 \times 10^3 \text{ rad/s} - 0}{3.2 \text{ s}} = \boxed{8.22 \times 10^2 \text{ rad/s}^2}$$

(b)

$$\theta_f = \omega_i t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}\left(8.22 \times 10^2 \text{ rad/s}^2\right)(3.2 \text{ s})^2 = \boxed{4.21 \times 10^3 \text{ rad}}$$

11. Make an order-of-magnitude estimate of the number of revolutions through which a typical automobile tire turns in 1 yr. State the quantities you measure or estimate and their values.

Estimate the tire's radius at 0.250 m and miles driven as 10 000 per year.

$$\theta = \frac{s}{r} = \frac{1.00 \times 10^4 \text{ mi}}{0.250 \text{ m}}\left(\frac{1\,609 \text{ m}}{1 \text{ mi}}\right) = 6.44 \times 10^7 \text{ rad/yr}$$

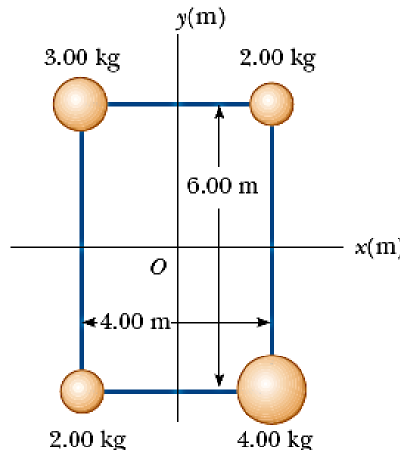
$$\theta = 6.44 \times 10^7 \text{ rad/yr}\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = 1.02 \times 10^7 \text{ rev/yr or } \boxed{\sim 10^7 \text{ rev/yr}}$$

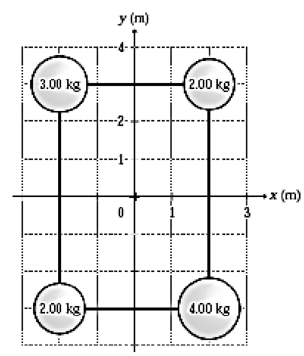
12. A racing car travels on a circular track of radius 250 m. If the car moves with a constant linear speed of 45.0 m/s, find (a) its angular speed and (b) the magnitude and direction of its acceleration.

(a) $$v = r\omega;\ \omega = \frac{v}{r} = \frac{45.0 \text{ m/s}}{250 \text{ m}} = \boxed{0.180 \text{ rad/s}}$$

(b) $$a_r = \frac{v^2}{r} = \frac{(45.0 \text{ m/s})^2}{250 \text{ m}} = \boxed{8.10 \text{ m/s}^2 \text{ toward the center of track}}$$

21. The four particles in Figure P10.21 are connected by rigid rods of negligible mass. The origin is at the center of the rectangle. If the system rotates in the xy plane about the z axis with an angular speed of 6.00 rad/s, calculate (a) the moment of inertia of the system about the z axis and (b) the rotational kinetic energy of the system.





(a) $$I = \sum_j m_j r_j^2$$

In this case,

$$r_1 = r_2 = r_3 = r_4$$

$$r = \sqrt{(3.00\text{ m})^2 + (2.00\text{ m})^2} = \sqrt{13.0}\text{ m}$$

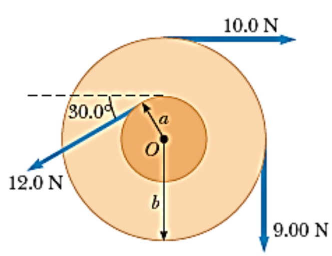
$$I = \left[\sqrt{13.0}\text{ m}\right]^2 [3.00 + 2.00 + 2.00 + 4.00]\text{ kg}$$

$$= \boxed{143\text{ kg}\cdot\text{m}^2}$$

(b) $$K_R = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(143\text{ kg}\cdot\text{m}^2\right)(6.00\text{ rad/s})^2$$

$$= \boxed{2.57 \times 10^3\text{ J}}$$

31. Find the net torque on the wheel in Figure P10.31 about the axle through O if a = 10.0 cm and b = 25.0 cm.



$$\sum \tau = 0.100\text{ m}(12.0\text{ N}) - 0.250\text{ m}(9.00\text{ N}) - 0.250\text{ m}(10.0\text{ N}) = \boxed{-3.55\text{ N}\cdot\text{m}}$$

The thirty-degree angle is unnecessary information.