Chapter 11

$\boldsymbol{\tau} \equiv \mathbf{r} \times \mathbf{F}$	
$\boldsymbol{\tau} = \frac{d\mathbf{L}}{dt}$	
$\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0$ $\hat{\mathbf{i}} \times \hat{\mathbf{j}} = -\hat{\mathbf{j}} \times \hat{\mathbf{i}} = \hat{\mathbf{k}}$ $\hat{\mathbf{j}} \times \hat{\mathbf{k}} = -\hat{\mathbf{k}} \times \hat{\mathbf{j}} = \hat{\mathbf{i}}$ $\hat{\mathbf{k}} \times \hat{\mathbf{i}} = -\hat{\mathbf{i}} \times \hat{\mathbf{k}} = \hat{\mathbf{j}}$	
$\mathbf{L} \equiv \mathbf{r} \times \mathbf{p}$	
$L = mvr \sin \phi$	
$L_z = I\omega$	
$\sum \tau_{\text{ext}} = I\alpha$	
$\mathbf{L}_i = \mathbf{L}_f$	

1. Given $\mathbf{M} = 6\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\mathbf{N} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$, calculate the vector product $\mathbf{M} \times \mathbf{N}$.

$$\mathbf{M}\times\mathbf{N} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 6 & 2 & -1 \\ 2 & -1 & -3 \end{vmatrix} = \boxed{-7.00\hat{\mathbf{i}} + 16.0\hat{\mathbf{j}} - 10.0\hat{\mathbf{k}}}$$

7. If $|\mathbf{A} \times \mathbf{B}| = \mathbf{A}\cdot\mathbf{B}$, what is the angle between $\mathbf{A}$ and $\mathbf{B}$?

$$|\mathbf{A}\times\mathbf{B}| = \mathbf{A}\cdot\mathbf{B} \Rightarrow AB\sin\theta = AB\cos\theta \Rightarrow \tan\theta = 1 \text{ or}$$

$$\theta = \boxed{45.0°}$$

8. A particle is located at the vector position $\mathbf{r} = (\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$ m, and the force acting on it is $\mathbf{F} = (3\hat{\mathbf{i}} + 2\hat{\mathbf{j}})$ N. What is the torque about (a) the origin and (b) the point having coordinates (0, 6) m?

(a) $$\tau = \mathbf{r}\times\mathbf{F} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 3 & 0 \\ 3 & 2 & 0 \end{vmatrix} = \hat{\mathbf{i}}(0-0) - \hat{\mathbf{j}}(0-0) + \hat{\mathbf{k}}(2-9) = \boxed{(-7.00\text{ N}\cdot\text{m})\hat{\mathbf{k}}}$$

(b) The particle's position vector relative to the new axis is $1\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 6\hat{\mathbf{j}} = 1\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$.

$$\tau = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & -3 & 0 \\ 3 & 2 & 0 \end{vmatrix} = \boxed{(11.0\text{ N}\cdot\text{m})\hat{\mathbf{k}}}$$

13. The position vector of a particle of mass 2.00 kg is given as a function of time by $\mathbf{r} = (6.00\hat{\mathbf{i}} + 5.00t\hat{\mathbf{j}})$ m. Determine the angular momentum of the particle about the origin, as a function of time.

$$\mathbf{r} = \left(6.00\hat{\mathbf{i}} + 5.00t\hat{\mathbf{j}}\ \text{m}\right) \qquad \mathbf{v} = \frac{d\mathbf{r}}{dt} = 5.00\hat{\mathbf{j}}\ \text{m/s}$$

so $\mathbf{p} = m\mathbf{v} = 2.00\ \text{kg}\left(5.00\hat{\mathbf{j}}\ \text{m/s}\right) = 10.0\hat{\mathbf{j}}\ \text{kg}\cdot\text{m/s}$

and $$\mathbf{L} = \mathbf{r}\times\mathbf{p} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 6.00 & 5.00t & 0 \\ 0 & 10.0 & 0 \end{vmatrix} = \boxed{\left(60.0\ \text{kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{k}}}$$

22. A uniform solid sphere of radius 0.500 m and mass 15.0 kg turns counterclockwise about a vertical axis through its center. Find its vector angular momentum when its angular speed is 3.00 rad/s.

The moment of inertia of the sphere about an axis through its center is

$$I = \frac{2}{5}MR^2 = \frac{2}{5}(15.0\ \text{kg})(0.500\ \text{m})^2 = 1.50\ \text{kg}\cdot\text{m}^2$$

Therefore, the magnitude of the angular momentum is

$$L = I\omega = \left(1.50\ \text{kg}\cdot\text{m}^2\right)(3.00\ \text{rad/s}) = 4.50\ \text{kg}\cdot\text{m}^2/\text{s}$$

Since the sphere rotates counterclockwise about the vertical axis, the angular momentum vector is directed upward in the $+z$ direction.

Thus, $\boxed{\mathbf{L} = \left(4.50\ \text{kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{k}}}$.