Chapter 15

$T = \frac{2\pi}{\omega}$	$v_{max} = \omega A = \sqrt{\frac{k}{m}} A$
$\omega = \sqrt{\frac{k}{m}}$	$a_{max} = \omega^2 A = \frac{k}{m} A$
$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$	

1. A ball dropped from a height of 4.00 m makes a perfectly elastic collision with the ground. Assuming no mechanical energy is lost due to air resistance, (a) show that the ensuing motion is periodic and (b) determine the period of the motion. (c) Is the motion simple harmonic? Explain.

(a) Since the collision is perfectly elastic, the ball will rebound to the height of 4.00 m and then repeat the motion over and over again. Thus, the $\boxed{\text{motion is periodic}}$.

(b) To determine the period, we use: $x = \frac{1}{2}gt^2$.

The time for the ball to hit the ground is $t = \sqrt{\frac{2x}{g}} = \sqrt{\frac{2(4.00\text{ m})}{9.80\text{ m/s}^2}} = 0.909\text{ s}$

This equals one-half the period, so $T = 2(0.909\text{ s}) = \boxed{1.82\text{ s}}$.

(c) $\boxed{\text{No}}$. The net force acting on the ball is a constant given by $F = -mg$ (except when it is in contact with the ground), which is not in the form of Hooke's law.

2. In an engine, a piston oscillates with simple harmonic motion so that its position varies according to the expression

$$x = (5.00 \text{ cm})\cos(2t + \pi/6)$$

where x is in centimeters and t is in seconds. At $t = 0$, find (a) the position of the piston, (b) its velocity, and (c) its acceleration. (d) Find the period and amplitude of the motion.

(a) $x = (5.00 \text{ cm})\cos\left(2t + \frac{\pi}{6}\right)$ At $t = 0$, $x = (5.00 \text{ cm})\cos\left(\frac{\pi}{6}\right) = \boxed{4.33 \text{ cm}}$

(b) $v = \frac{dx}{dt} = -(10.0 \text{ cm/s})\sin\left(2t + \frac{\pi}{6}\right)$ At $t = 0$, $v = \boxed{-5.00 \text{ cm/s}}$

(c) $a = \frac{dv}{dt} = -\left(20.0 \text{ cm/s}^2\right)\cos\left(2t + \frac{\pi}{6}\right)$ At $t = 0$, $a = \boxed{-17.3 \text{ cm/s}^2}$

(d) $A = \boxed{5.00 \text{ cm}}$ and $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \boxed{3.14 \text{ s}}$

7. A simple harmonic oscillator takes 12.0 s to undergo five complete vibrations. Find (a) the period of its motion, (b) the frequency in hertz, and (c) the angular frequency in radians per second.

(a) $T = \frac{12.0 \text{ s}}{5} = \boxed{2.40 \text{ s}}$

(b) $f = \frac{1}{T} = \frac{1}{2.40} = \boxed{0.417 \text{ Hz}}$

(c) $\omega = 2\pi f = 2\pi(0.417) = \boxed{2.62 \text{ rad/s}}$

9. A 7.00-kg object is hung from the bottom end of a vertical spring fastened to an overhead beam. The object is set into vertical oscillations having a period of 2.60 s. Find the force constant of the spring.

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad \text{or}$$

Solving for k,

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}}$$

$$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2 (7.00\ \text{kg})}{(2.60\ \text{s})^2} = \boxed{40.9\ \text{N/m}}.$$

13. A 1.00-kg object is attached to a horizontal spring. The spring is initially stretched by 0.100 m, and the object is released from rest there. It proceeds to move without friction. The next time the speed of the object is zero is 0.500 s later. What is the maximum speed of the object?

The 0.500 s must elapse between one turning point and the other.

Thus the period is 1.00 s.

$$\omega = \frac{2\pi}{T} = 6.28/\text{s}$$

and $v_{\max} = \omega A = (6.28/\text{s})(0.100\ \text{m}) = \boxed{0.628\ \text{m/s}}$.

27. A man enters a tall tower, needing to know its height. He notes that a long pendulum extends from the ceiling almost to the floor and that its period is 12.0 s. (a) How tall is the tower? (b) **What If?** If this pendulum is taken to the Moon, where the free-fall acceleration is 1.67 m/s^2, what is its period there?

(a)

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$L = \frac{gT^2}{4\pi^2} = \frac{(9.80\ \text{m/s}^2)(12.0\ \text{s})^2}{4\pi^2} = \boxed{35.7\ \text{m}}$$

(b)

$$T_{\text{moon}} = 2\pi\sqrt{\frac{L}{g_{\text{moon}}}} = 2\pi\sqrt{\frac{35.7\ \text{m}}{1.67\ \text{m/s}^2}} = \boxed{29.1\ \text{s}}$$