Chapter 4

$\vec{\mathbf{v}}_{avg} \equiv \frac{\Delta \vec{\mathbf{r}}}{\Delta t}$	$a_c = \frac{v^2}{r}$
$\vec{\mathbf{a}}_{avg} \equiv \frac{\Delta \vec{\mathbf{v}}}{\Delta t} = \frac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{t_f - t_i}$	$T = \frac{2\pi r}{v}$
$\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$	$\vec{\mathbf{a}} = \vec{\mathbf{a}}_r + \vec{\mathbf{a}}_t$
$v_{xi} = v_i \cos \theta_i$	$a_t = \left\| \frac{dv}{dt} \right\|$
$v_{yi} = v_i \sin \theta_i$	$a_r = -a_c = -\frac{v^2}{r}$
$t_{Ⓐ} = \frac{v_i \sin \theta_i}{g}$	$a = \sqrt{a_r^2 + a_t^2}.$
$h = \frac{v_i^2 \sin^2 \theta_i}{2g}$	
$R = \frac{v_i^2 \sin 2\theta_i}{g}$	
$R_{max} = v_i^2 / g.$	

Example 4.6

What is the centripetal acceleration of the Earth as it moves in its orbit around the Sun?

Radius of the Earth's orbit around the Sun = r = 1.496×10^{11} m

$$T = \frac{2\pi r}{v}$$

$$v = \frac{2\pi r}{T}$$

$$a_c = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2}$$

$$a_c = \frac{4\pi^2 (1.496 \times 10^{11}\ \text{m})}{(1\ \text{yr})^2}\left(\frac{1\ \text{yr}}{3.156 \times 10^7\ \text{s}}\right)^2 = 5.93 \times 10^{-3}\ \text{m/s}^2$$

2. When the Sun is directly overhead, a hawk dives toward the ground with a constant velocity of 5.00 m/s at 60.0° below the horizontal. Calculate the speed of its shadow on the level ground.

Solution:

The sun projects onto the ground the x-component of her velocity:

$$5.00 \text{ m/s } \cos(-60.0°) = \boxed{2.50 \text{ m/s}}.$$

5. The vector position of a particle varies in time according to the expression $\vec{\mathbf{r}} = 3.00\hat{\mathbf{i}} - 6.00t^2\hat{\mathbf{j}}$, where $\vec{\mathbf{r}}$ is in meters and t is in seconds. (a) Find an expression for the velocity of the particle as a function of time. (b) Determine the acceleration of the particle as a function of time. (c) Calculate the particle's position and velocity at $t = 1.00$ s.

Solution:

(a)

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \left(\frac{d}{dt}\right)\left(3.00\hat{\mathbf{i}} - 6.00t^2\hat{\mathbf{j}}\right) = \boxed{-12.0t\,\hat{\mathbf{j}} \text{ m/s}}$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \left(\frac{d}{dt}\right)\left(-12.0t\,\hat{\mathbf{j}}\right) = \boxed{-12.0\hat{\mathbf{j}} \text{ m/s}^2}$$

(b)

$$\boxed{\mathbf{r} = \left(3.00\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}}\right) \text{ m; } \mathbf{v} = -12.0\hat{\mathbf{j}} \text{ m/s}}$$

11. A projectile is fired in such a way that its horizontal range is equal to three times its maximumv height. What is the angle of projection?

Solution:

$$h = \frac{v_i^2 \sin^2 \theta_i}{2g};\ R = \frac{v_i^2 (\sin 2\theta_i)}{g};$$

$$3h = R,$$

$$\text{so } \frac{3v_i^2 \sin^2 \theta_i}{2g} = \frac{v_i^2 (\sin 2\theta_i)}{g}$$

$$\text{or } \frac{2}{3} = \frac{\sin^2 \theta_i}{\sin 2\theta_i} = \frac{\tan \theta_i}{2}$$

$$\text{thus } \theta_i = \tan^{-1}\left(\frac{4}{3}\right) = \boxed{53.1°}.$$

12. To start an avalanche on a mountain slope, an artillery shell is fired with an initial velocity of 300 m/s at 55.0° above the horizontal. It explodes on the mountainside 42.0 s after firing. What are the x and y coordinates of the shell where it explodes, relative to its firing point?

Solution:

$$x = v_{xi} t = v_i \cos \theta_i t$$

$$x = (300\ \text{m/s})(\cos 55.0°)(42.0\ \text{s})$$

$$x = \boxed{7.23 \times 10^3\ \text{m}}$$

$$y = v_{yi} t - \frac{1}{2} g t^2 = v_i \sin \theta_i t - \frac{1}{2} g t^2$$

$$y = (300\ \text{m/s})(\sin 55.0°)(42.0\ \text{s}) - \frac{1}{2}\left(9.80\ \text{m/s}^2\right)(42.0\ \text{s})^2$$

$$= \boxed{1.68 \times 10^3\ \text{m}}$$

16. A ball is tossed from an upper-story window of a building. The ball is given an initial velocity of 8.00 m/s at an angle of 20.0° below the horizontal. It strikes the ground 3.00 s later. (a) How far horizontally from the base of the building does the ball strike the ground? (b) Find the height from which the ball was thrown. (c) How long does it take the ball to reach a point 10.0 m below the level of launching?

Solution:

(a) $$x_f = v_{xi}t = 8.00\cos 20.0°(3.00) = \boxed{22.6\text{ m}}$$

(b) Taking y positive downwards,

$$y_f = v_{yi}t + \frac{1}{2}gt^2$$

$$y_f = 8.00\sin 20.0°(3.00) + \frac{1}{2}(9.80)(3.00)^2 = \boxed{52.3\text{ m}}.$$

(c) $$10.0 = 8.00(\sin 20.0°)t + \frac{1}{2}(9.80)t^2$$

$$4.90t^2 + 2.74t - 10.0 = 0$$

$$t = \frac{-2.74 \pm \sqrt{(2.74)^2 + 196}}{9.80} = \boxed{1.18\text{ s}}$$

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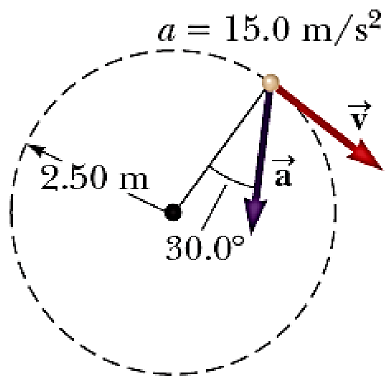
27. The athlete shown in Figure P4.27 rotates a 1.00-kg discus along a circular path of radius 1.06 m. The maximum speed of the discus is 20.0 m/s. Determine the magnitude of the maximum radial acceleration of the discus.

Solution:

$$a_c = \frac{v^2}{r} = \frac{(20.0\text{ m/s})^2}{1.06\text{ m}} = \boxed{377\text{ m/s}^2}$$

The mass is unnecessary information.

32. Figure P4.32 represents the total acceleration of a particle moving clockwise in a circle of radius 2.50 m at a certain instant of time. For that instant, find (a) the radial acceleration of the particle, (b) the speed of the particle, and (c) its tangential acceleration.



Solution:

$r = 2.50\text{ m}$, $a = 15.0\ \text{m/s}^2$

(a) $$a_c = a\cos 30.0^\circ = \left(15.0\ \text{m/s}^2\right)(\cos 30^\circ) = \boxed{13.0\ \text{m/s}^2}$$

(b) $$a_c = \frac{v^2}{r}$$

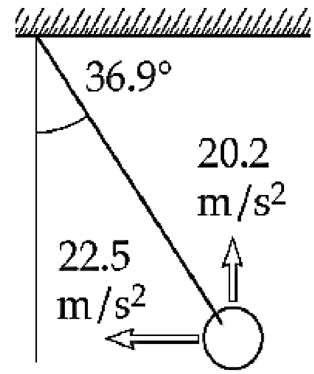
$$\text{so } v^2 = ra_c = 2.50\text{ m}\left(13.0\ \text{m/s}^2\right) = 32.5\ \text{m}^2/\text{s}^2$$

$$v = \sqrt{32.5}\ \text{m/s} = \boxed{5.70\ \text{m/s}}$$

(c) $$a^2 = a_t^2 + a_r^2$$

$$\text{so } a_t = \sqrt{a^2 - a_r^2} = \sqrt{\left(15.0\ \text{m/s}^2\right)^2 - \left(13.0\text{m/s}^2\right)} = \boxed{7.50\ \text{m/s}^2}$$

34. A ball swings in a vertical circle at the end of a rope 1.50 m long. When the ball is 36.9° past the lowest point on its way up, its total acceleration is $-22.5\hat{\mathbf{i}} + 20.2\hat{\mathbf{j}}$ m/s². For that instant, (a) sketch a vector diagram showing the components of its acceleration, (b) determine the magnitude of its radial acceleration, and (c) determine the speed and velocity of the ball.



Solution:

(a) See figure to the right.

(b) The components of the 20.2 and the 22.5 m/s^2 along the rope together constitute the centripetal acceleration:

$$a_c = \left(22.5\ \text{m/s}^2\right)\cos(90.0° - 36.9°) + \left(20.2\ \text{m/s}^2\right)\cos 36.9° = \boxed{29.7\ \text{m/s}^2}$$

(c) $a_c = \dfrac{v^2}{r}$ so $v = \sqrt{a_c r} = \sqrt{29.7\ \text{m/s}^2\,(1.50\ \text{m})} = 6.67\ \text{m/s}$ tangent to circle

$\mathbf{v} = \boxed{6.67\ \text{m/s at } 36.9° \text{ above the horizontal}}$