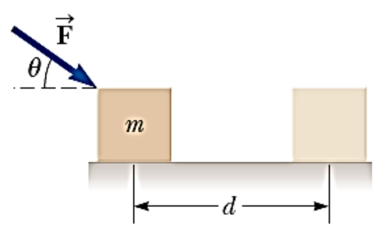
Chapter 7

$W = F\Delta r\cos\theta = \vec{\mathbf{F}}\cdot\Delta\vec{\mathbf{r}}$
$\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} \equiv AB\cos\theta$
$\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = 1$
$\hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{i}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = 0$
$F_s = -kx$
$W_{\text{ext}} = U_g \equiv mgy$
$W_{\text{ext}} = mgy_f - mgy_i$
$W_{\text{app}} = \frac{1}{2}kx_f^{\,2} - \frac{1}{2}kx_i^{\,2}$

1. A block of mass 2.50 kg is pushed 2.20 m along a frictionless horizontal table by a constant 16.0-N force directed 25.0° below the horizontal. Determine the work done on the block by (a) the applied force, (b) the normal force exerted by the table, and (c) the gravitational force. (d) Determine the total work done on the block.



Solution

(a) $W = F\Delta r\cos\theta = (16.0\text{ N})(2.20\text{ m})\cos 25.0° = \boxed{31.9\text{ J}}$

(b), (c) The normal force and the weight are both at 90° to the displacement in any time interval. Both do $\boxed{0}$ work.

(d) $\sum W = 31.9\text{ J} + 0 + 0 = \boxed{31.9\text{ J}}$

4. A raindrop of mass 3.35×10^{-5} kg falls vertically at constant speed under the influence of gravity and air resistance. Model the drop as a particle. As it falls 100 m, what is the work done on the raindrop (a) by the gravitational force and (b) by air resistance?

Solution

(a) $$W = mgh = \left(3.35 \times 10^{-5}\right)(9.80)(100) \text{ J} = \boxed{3.28 \times 10^{-2} \text{ J}}$$

(b) Since $R = mg$, $W_{\text{air resistance}} = \boxed{-3.28 \times 10^{-2} \text{ J}}$

5 A shopper in a supermarket pushes a cart with a force of 35.0 N directed at an angle of 25.0° downward from the horizontal. Find the work done by the shopper on the cart as he moves down an aisle 50.0 m long.

Solution

The component of force along the direction of motion is

$$F\cos\theta = (35.0 \text{ N})\cos 25.0° = 31.7 \text{ N}.$$

The work done by this force is

$$W = (F\cos\theta)\Delta r = (31.7 \text{ N})(50.0 \text{ m}) = \boxed{1.59 \times 10^{3} \text{ J}}.$$

5. Vector **A** has a magnitude of 5.00 units, and **B** has a magnitude of 9.00 units. The two vectors make an angle of 50.0° with each other. Find $\mathbf{A} \cdot \mathbf{B}$.

Solution

$$A = 5.00 \, ; \; B = 9.00 \, ; \; \theta = 50.0°$$

$$\mathbf{A} \cdot \mathbf{B} = AB\cos\theta = (5.00)(9.00)\cos 50.0° = \boxed{28.9}$$

7. A force $\mathbf{F} = (6\hat{\mathbf{i}} - 2\hat{\mathbf{j}})$ N acts on a particle that undergoes a displacement $\Delta\mathbf{r} = (3\hat{\mathbf{i}} + \hat{\mathbf{j}})$ m. Find (a) the work done by the force on the particle and (b) the angle between **F** and $\Delta\mathbf{r}$.

Solution

$$W = \mathbf{F} \cdot \Delta\mathbf{r} = F_x x + F_y y = (6.00)(3.00) \text{ N} \cdot \text{m} + (-2.00)(1.00) \text{ N} \cdot \text{m} = \boxed{16.0 \text{ J}}$$

$$\theta = \cos^{-1}\left(\frac{\mathbf{F} \cdot \Delta\mathbf{r}}{F\Delta r}\right) = \cos^{-1}\frac{16}{\sqrt{\left((6.00)^2 + (-2.00)^2\right)\left((3.00)^2 + (1.00)^2\right)}} = \boxed{36.9°}$$

10. For $\mathbf{A} = 3\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$, $\mathbf{B} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$, and $\mathbf{C} = 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$, find $\mathbf{C} \cdot (\mathbf{A} - \mathbf{B})$.

Solution

$$\mathbf{A} - \mathbf{B} = \left(3.00\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}\right) - \left(-\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}} + 5.00\hat{\mathbf{k}}\right)$$

$$\mathbf{A} - \mathbf{B} = 4.00\hat{\mathbf{i}} - \hat{\mathbf{j}} - 6.00\hat{\mathbf{k}}$$

$$\mathbf{C} \cdot (\mathbf{A} - \mathbf{B}) = \left(2.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{k}}\right) \cdot \left(4.00\hat{\mathbf{i}} - \hat{\mathbf{j}} - 6.00\hat{\mathbf{k}}\right) = 0 + (-2.00) + (+18.0) = \boxed{16.0}$$

9. Using the definition of the scalar product, find the angles between (a) $\mathbf{A} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$ and $\mathbf{B} = 4\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$; (b) $\mathbf{A} = -2\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$ and $\mathbf{B} = 3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$; (c) $\mathbf{A} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\mathbf{B} = 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$.

(a)

$$\mathbf{A} = 3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}$$

$$\mathbf{B} = 4.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}}$$

$$\theta = \cos^{-1}\frac{\mathbf{A}\cdot\mathbf{B}}{AB} = \cos^{-1}\frac{12.0 + 8.00}{\sqrt{(13.0)(32.0)}} = \boxed{11.3^\circ}$$

(b)

$$\mathbf{B} = 3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}} + 2.00\hat{\mathbf{k}}$$

$$\mathbf{A} = -2.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}$$

$$\cos\theta = \frac{\mathbf{A}\cdot\mathbf{B}}{AB} = \frac{-6.00 - 16.0}{\sqrt{(20.0)(29.0)}} = \boxed{156^\circ}$$

(c)

$$\mathbf{A} = \hat{\mathbf{i}} - 2.00\hat{\mathbf{j}} + 2.00\hat{\mathbf{k}}$$

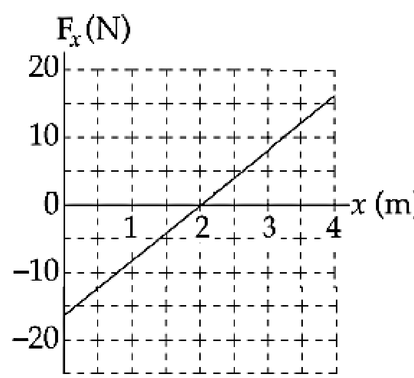
$$\mathbf{B} = 3.00\hat{\mathbf{j}} + 4.00\hat{\mathbf{k}}$$

$$\theta = \cos^{-1}\left(\frac{\mathbf{A}\cdot\mathbf{B}}{AB}\right) = \cos^{-1}\left(\frac{-6.00 + 8.00}{\sqrt{9.00}\cdot\sqrt{25.0}}\right) = \boxed{82.3^\circ}$$

12. The force acting on a particle is $F_x = (8x - 16)$ N, where x is in meters. (a) Make a plot of this force versus x from $x = 0$ to $x = 3.00$ m. (b) From your graph, find the net work done by this force on the particle as it moves from $x = 0$ to $x = 3.00$ m.

Solution

$$F_x = (8x - 16)\text{ N}$$



(a) See figure to the right

(b)

$$W_{\text{net}} = \frac{-(2.00\text{ m})(16.0\text{ N})}{2} + \frac{(1.00\text{ m})(8.00\text{ N})}{2} = \boxed{-12.0\text{ J}}$$

24. A 0.600-kg particle has a speed of 2.00 m/s at point Ⓐ and kinetic energy of 7.50 J at point Ⓑ. What is (a) its kinetic energy at Ⓐ? (b) its speed at Ⓑ? (c) the total work done on the particle as it moves from Ⓐ to Ⓑ?

Solution

(a) $$K_A = \frac{1}{2}(0.600\text{ kg})(2.00\text{ m/s})^2 = \boxed{1.20\text{ J}}$$

(b) $$\frac{1}{2}mv_B^2 = K_B:\ v_B = \sqrt{\frac{2K_B}{m}} = \sqrt{\frac{(2)(7.50)}{0.600}} = \boxed{5.00\text{ m/s}}$$

(c) $$\sum W = \Delta K = K_B - K_A = \frac{1}{2}m\left(v_B^2 - v_A^2\right) = 7.50\text{ J} - 1.20\text{ J} = \boxed{6.30\text{ J}}$$

26. A 3.00-kg object has a velocity $(6.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}})$ m/s. (a) What is its kinetic energy at this time? (b) Find the total work done on the object if its velocity changes to $(8.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}})$ m/s. (*Note:* From the definition of the dot product, $v^2 = \mathbf{v}\cdot\mathbf{v}$.)

Solution

$$\mathbf{v}_i = \left(6.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}\right) = \text{ m/s}$$

(a) $$v_i = \sqrt{v_{ix}^2 + v_{iy}^2} = \sqrt{40.0}\text{ m/s}$$

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(3.00\text{ kg})\left(40.0\text{ m}^2/\text{s}^2\right) = \boxed{60.0\text{ J}}$$

(b) $$\mathbf{v}_f = 8.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}$$

$$v_f^2 = \mathbf{v}_f\cdot\mathbf{v}_f = 64.0 + 16.0 = 80.0\text{ m}^2/\text{s}^2$$

$$\Delta K = K_f - K_i = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = \frac{3.00}{2}(80.0) - 60.0 = \boxed{60.0\text{ J}}$$