Chapter9

$\vec{\mathbf{p}} \equiv m\vec{\mathbf{v}}$
$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}} = m\frac{d\vec{\mathbf{v}}}{dt}$
$\sum \vec{\mathbf{F}} = \frac{d\ m\vec{\mathbf{v}}}{dt} = \frac{d\vec{\mathbf{p}}}{dt}$
$\vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i} = \vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f}$
$m_1v_{1_i} + m_2v_{2_i} = m_1v_{1_f} + m_2v_{2f}$
$\frac{1}{2}m_1{v_{1_i}}^2 + \frac{1}{2}m_2{v_{2_i}}^2 = \frac{1}{2}m_1{v_{1_f}}^2 + \frac{1}{2}m_2{v_{2_f}}^2$
$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2i}$
$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_{2i}$
$x_{CM} \equiv \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$
$y_{CM} \equiv \frac{1}{M}\sum_i m_iy_i$
$z_{CM} \equiv \frac{1}{M}\sum_i m_iz_i$

1. A 3.00-kg particle has a velocity of $(3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}})$ m/s. (a) Find its x and y components of momentum. (b) Find the magnitude and direction of its momentum.

P9.1 $m = 3.00$ kg, $\mathbf{v} = (3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}})$ m/s

(a) $\mathbf{p} = m\mathbf{v} = (9.00\hat{\mathbf{i}} - 12.0\hat{\mathbf{j}})$ kg·m/s

Thus, $\boxed{p_x = 9.00 \text{ kg}\cdot\text{m/s}}$

and $\boxed{p_y = -12.0 \text{ kg}\cdot\text{m/s}}$

(b) $p = \sqrt{p_x^2 + p_y^2} = \sqrt{(9.00)^2 + (12.0)^2} = \boxed{15.0 \text{ kg}\cdot\text{m/s}}$

$\theta = \tan^{-1}\left(\frac{p_y}{p_x}\right) = \tan^{-1}(-1.33) = \boxed{307°}$

4. Two blocks of masses M and $3M$ are placed on a horizontal, frictionless surface. A light spring is attached to one of them, and the blocks are pushed together with the spring between them (Fig. P9.4). A cord initially holding the blocks together is burned; after this, the block of mass $3M$ moves to the right with a speed of 2.00 m/s. (a) What is the speed of the block of mass M? (b) Find the original elastic potential energy in the spring if $M = 0.350$ kg.

(a) For the system of two blocks $\Delta p = 0$,

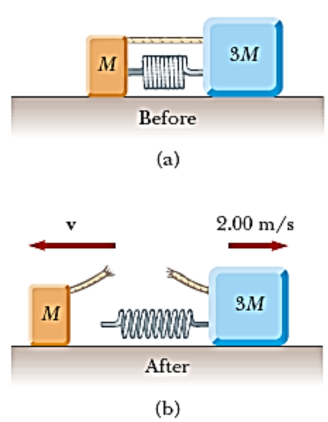
or $p_i = p_f$

Therefore, $0 = Mv_m + (3M)(2.00 \text{ m/s})$

Solving gives $v_m = \boxed{-6.00 \text{ m/s}}$

(motion toward the left).

(b) $\frac{1}{2}kx^2 = \frac{1}{2}Mv_M^2 + \frac{1}{2}(3M)v_{3M}^2 = \boxed{8.40 \text{ J}}$



6. A friend claims that, as long as he has his seatbelt on, he can hold on to a 12.0-kg child in a 60.0 mi/h head-on collision with a brick wall in which the car passenger compartment comes to a stop in 0.050 0 s. Show that the violent force during the collision will tear the child from his arms. A child should always be in a toddler seat secured with a seat belt in the back seat of a car.

From the impulse-momentum theorem, $\overline{F}(\Delta t) = \Delta p = mv_f - mv_i$, the average force required to hold onto the child is

$$\overline{F} = \frac{m(v_f - v_i)}{(\Delta t)} = \frac{(12\text{ kg})(0 - 60\text{ mi/h})}{0.050\text{ s} - 0}\left(\frac{1\text{ m/s}}{2.237\text{ mi/h}}\right) = -6.44 \times 10^3\text{ N}.$$

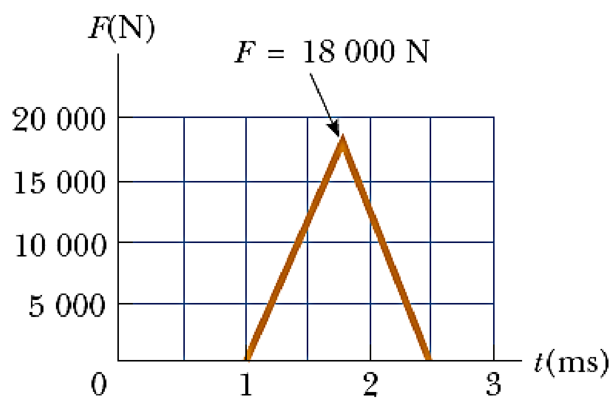
Therefore, the magnitude of the needed retarding force is $\boxed{6.44 \times 10^3\text{ N}}$, or 1 400 lb. A person cannot exert a force of this magnitude and a safety device should be used.

7. An estimated force–time curve for a baseball struck by a bat is shown in Figure P9.7. From this curve, determine (a) the impulse delivered to the ball, (b) the average force exerted on the ball, and (c) the peak force exerted on the ball.

(a) $I = \int F dt =$ area under curve

$$I = \frac{1}{2}(1.50 \times 10^{-3}\text{ s})(18\,000\text{ N})$$

$$= \boxed{13.5\text{ N}\cdot\text{s}}$$



(b) $$F = \frac{13.5\text{ N}\cdot\text{s}}{1.50 \times 10^{-3}\text{ s}} = \boxed{9.00\text{ kN}}$$

(c) From the graph, we see that $F_{max} = \boxed{18.0\text{ kN}}$

15. High-speed stroboscopic photographs show that the head of a golf club of mass 200 g is traveling at 55.0 m/s just before it strikes a 46.0-g golf ball at rest on a tee. After the collision, the club head travels (in the same direction) at 40.0 m/s. Find the speed of the golf ball just after impact.

$$(200\ \text{g})(55.0\ \text{m/s}) = (46.0\ \text{g})v + (200\ \text{g})(40.0\ \text{m/s})$$

$$v = \boxed{65.2\ \text{m/s}}$$

18. A railroad car of mass 2.50×10^4 kg is moving with a speed of 4.00 m/s. It collides and couples with three other coupled railroad cars, each of the same mass as the single car and moving in the same direction with an initial speed of 2.00 m/s. (a) What is the speed of the four cars after the collision? (b) How much mechanical energy is lost in the collision?

(a) $$mv_{1i} + 3mv_{2i} = 4mv_f \quad \text{where } m = 2.50 \times 10^4\ \text{kg}$$

$$v_f = \frac{4.00 + 3(2.00)}{4} = \boxed{2.50\ \text{m/s}}$$

(b) $$K_f - K_i = \frac{1}{2}(4m)v_f^2 - \left[\frac{1}{2}mv_{1i}^2 + \frac{1}{2}(3m)v_{2i}^2\right]$$

$$= \left(2.50 \times 10^4\right)(12.5 - 8.00 - 6.00) = \boxed{-3.75 \times 10^4\ \text{J}}$$

38. Four objects are situated along the y axis as follows: a 2.00 kg object is at $+3.00$ m, a 3.00-kg object is at $+2.50$ m, a 2.50-kg object is at the origin, and a 4.00-kg object is at -0.500 m. Where is the center of mass of these objects?

The x-coordinate of the center of mass is

$$x_{CM} = \frac{\sum m_i x_i}{\sum m_i} = \frac{0+0+0+0}{(2.00\text{ kg} + 3.00\text{ kg} + 2.50\text{ kg} + 4.00\text{ kg})}$$

$$\boxed{x_{CM} = 0}$$

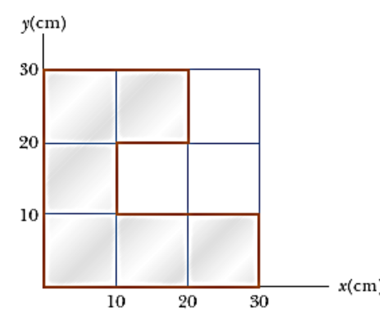
and the y-coordinate of the center of mass is

$$y_{CM} = \frac{\sum m_i y_i}{\sum m_i} = \frac{(2.00\text{ kg})(3.00\text{ m}) + (3.00\text{ kg})(2.50\text{ m}) + (2.50\text{ kg})(0) + (4.00\text{ kg})(-0.500\text{ m})}{2.00\text{ kg} + 3.00\text{ kg} + 2.50\text{ kg} + 4.00\text{ kg}}$$

$$\boxed{y_{CM} = 1.00\text{ m}}$$

41. A uniform piece of sheet steel is shaped as in Figure P9.41. Compute the x and y coordinates of the center of mass of the piece.

Let A_1 represent the area of the bottom row of squares, A_2 the middle square, and A_3 the top pair.



$$A = A_1 + A_2 + A_3$$

$$M = M_1 + M_2 + M_3$$

$$\frac{M_1}{A_1} = \frac{M}{A}$$

$$A_1 = 300\text{ cm}^2,\ A_2 = 100\text{ cm}^2,\ A_3 = 200\text{ cm}^2,\ A = 600\text{ cm}^2$$

$$M_1 = M\left(\frac{A_1}{A}\right) = \frac{300\text{ cm}^2}{600\text{ cm}^2}M = \frac{M}{2}$$

$$M_2 = M\left(\frac{A_2}{A}\right) = \frac{100\text{ cm}^2}{600\text{ cm}^2}M = \frac{M}{6}$$

$$M_3 = M\left(\frac{A_3}{A}\right) = \frac{200\text{ cm}^2}{600\text{ cm}^2}M = \frac{M}{3}$$

$$x_{CM} = \frac{x_1M_1 + x_2M_2 + x_3M_3}{M} = \frac{15.0\text{ cm}\left(\frac{1}{2}M\right) + 5.00\text{ cm}\left(\frac{1}{6}M\right) + 10.0\text{ cm}\left(\frac{1}{3}M\right)}{M}$$

$$x_{CM} = \boxed{11.7\text{ cm}}$$

$$y_{CM} = \frac{\frac{1}{2}M(5.00\text{ cm}) + \frac{1}{6}M(15.0\text{ cm}) + \left(\frac{1}{3}M\right)(25.0\text{ cm})}{M} = 13.3\text{ cm}$$

$$y_{CM} = \boxed{13.3\text{ cm}}$$

45. A 2.00-kg particle has a velocity $(2.00\hat{\mathbf{i}} - 3.00\hat{\mathbf{j}})$ m/s, and a 3.00-kg particle has a velocity $(1.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}})$ m/s. Find (a) the velocity of the center of mass and (b) the total momentum of the system.

(a) $$\mathbf{v}_{\text{CM}} = \frac{\sum m_i \mathbf{v}_i}{M} = \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{M}$$

$$= \frac{(2.00\text{ kg})\left(2.00\hat{\mathbf{i}}\text{ m/s} - 3.00\hat{\mathbf{j}}\text{ m/s}\right) + (3.00\text{ kg})\left(1.00\hat{\mathbf{i}}\text{ m/s} + 6.00\hat{\mathbf{j}}\text{ m/s}\right)}{5.00\text{ kg}}$$

$$\mathbf{v}_{\text{CM}} = \boxed{\left(1.40\hat{\mathbf{i}} + 2.40\hat{\mathbf{j}}\right)\text{ m/s}}$$

(b) $$\mathbf{p} = M\mathbf{v}_{\text{CM}} = (5.00\text{ kg})\left(1.40\hat{\mathbf{i}} + 2.40\hat{\mathbf{j}}\right)\text{ m/s} = \boxed{\left(7.00\hat{\mathbf{i}} + 12.0\hat{\mathbf{j}}\right)\text{ kg}\cdot\text{m/s}}$$