Chapter 7

Kinetic Energy and Work

In this chapter we will introduce the following concepts:

7.2	Work Done by a Constant Force
7.3	The Scalar Product of Two Vectors
7.4	Work Done by a Varying Force
7.5	Kinetic Energy and the Work–Kinetic Energy Theorem
7.6	Potential Energy of a System
7.7	Conservative and Non conservative Forces
7.8	Relation B/w Conservative Forces and Potential energy

Work Done by a Constant Force

The **work** W done on a system by an agent exerting a constant force on the system is the product of the magnitude F of the force, the magnitude Δr of the displacement of the point of application of the force, and $\cos\theta$, where θ is the angle between the force and displacement vectors:

$$W \equiv F\,\Delta r\cos\theta \tag{7.1}$$

If an applied force $\vec{\mathbf{F}}$ is in the same direction as the displacement $\Delta\vec{\mathbf{r}}$, then $\theta = 0$ and $\cos 0 = 1$. In this case, Equation 7.1 gives

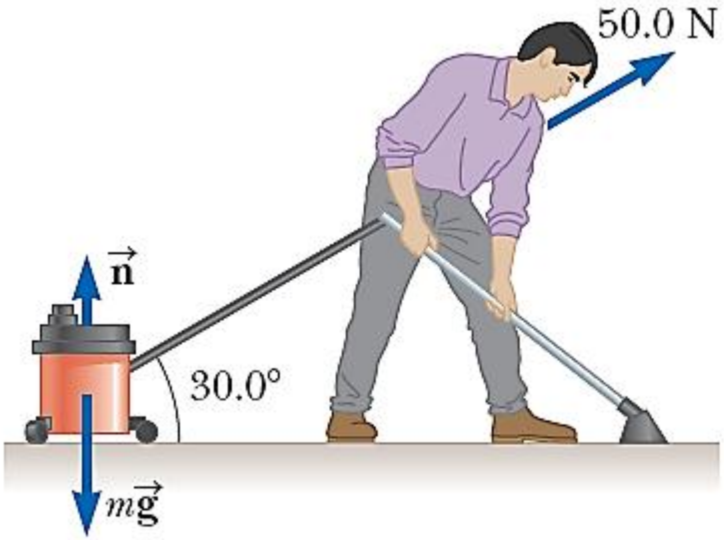
$$W = F\,\Delta r$$

The units of work are those of force multiplied by those of length. Therefore, the SI unit of work is the **newton · meter** ($\mathrm{N \cdot m = kg \cdot m^2/s^2}$). This combination of units is used so frequently that it has been given a name of its own, the **joule** (J).

Example 7.1 Mr. Clean

A man cleaning a floor pulls a vacuum cleaner with a force of magnitude $F = 50.0\ \mathrm{N}$ at an angle of 30.0° with the horizontal (Fig. 7.5). Calculate the work done by the force on the vacuum cleaner as the vacuum cleaner is displaced 3.00 m to the right.

$$W = F\Delta r\cos\theta = (50.0\ \mathrm{N})(3.00\ \mathrm{m})(\cos 30.0^\circ)$$
$$= \boxed{130\ \mathrm{J}}$$



The Scalar Product of Two Vectors

The scalar product of any two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is defined as a scalar quantity equal to the product of the magnitudes of the two vectors and the cosine of the angle θ between them:

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} \equiv AB\cos\theta \qquad (7.2)$$

In other words, $\vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}}$ is a shorthand notation for $F\Delta r\cos\theta$.

$$W = F\Delta r\cos\theta = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}} \qquad (7.3)$$

$$\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{k}} = 1$$

$$\hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{i}} \cdot \hat{\mathbf{k}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{k}} = 0$$

Example 7.2 The Scalar Product

The vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are given by $\vec{\mathbf{A}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ and $\vec{\mathbf{B}} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$.

(A) Determine the scalar product $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$.

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = \quad 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} \quad \cdot \quad -\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$$

$$= -2\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} + 2\hat{\mathbf{i}} \cdot 2\hat{\mathbf{j}} - 3\hat{\mathbf{j}} \cdot \hat{\mathbf{i}} + 3\hat{\mathbf{j}} \cdot 2\hat{\mathbf{j}}$$

$$= -2(1) + 4(0) - 3(0) + 6(1) = -2 + 6 = \boxed{4}$$

$A_x = 2$, $A_y = 3$, $B_x = -1$, and $B_y = 2$.

(B) Find the angle θ between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$.

$$A = \sqrt{A_x^{\,2} + A_y^{\,2}} = \sqrt{\ 2\ ^2 + \ 3\ ^2} = \sqrt{13}$$

$$B = \sqrt{B_x^{\,2} + B_y^{\,2}} = \sqrt{\ -1\ ^2 + \ 2\ ^2} = \sqrt{5}$$

$$\cos\theta = \frac{\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}}{AB} = \frac{4}{\sqrt{13}\sqrt{5}} = \frac{4}{\sqrt{65}}$$

$$\theta = \cos^{-1}\frac{4}{\sqrt{65}} = \boxed{60.3^\circ}$$

Example 7.3 Work Done by a Constant Force

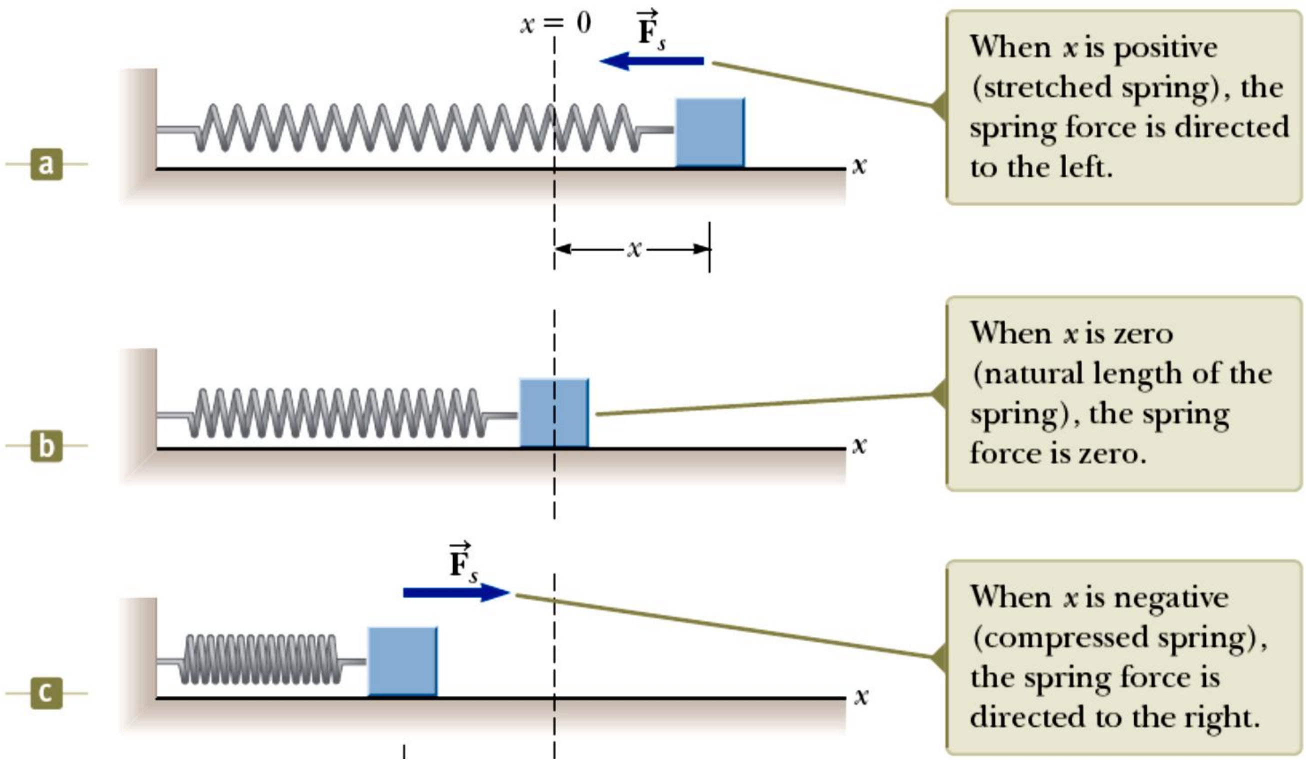
A particle moving in the *xy* plane undergoes a displacement given by $\Delta\vec{\mathbf{r}} = (2.0\hat{\mathbf{i}} + 3.0\hat{\mathbf{j}})$ m as a constant force $\vec{\mathbf{F}} = (5.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}})$ N acts on the particle. Calculate the work done by $\vec{\mathbf{F}}$ on the particle.

$$W = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}} = [(5.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}})\text{ N}] \cdot [(2.0\hat{\mathbf{i}} + 3.0\hat{\mathbf{j}})\text{ m}]$$

$$= (5.0\hat{\mathbf{i}} \cdot 2.0\hat{\mathbf{i}} + 5.0\hat{\mathbf{i}} \cdot 3.0\hat{\mathbf{j}} + 2.0\hat{\mathbf{j}} \cdot 2.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}} \cdot 3.0\hat{\mathbf{j}})\text{ N} \cdot \text{m}$$

$$= [10 + 0 + 0 + 6]\text{ N} \cdot \text{m} = \boxed{16\text{ J}}$$

Work Done by a Spring



$$F_s = -kx \quad (7.9)$$

where x is the position of the block relative to its equilibrium ($x = 0$) position and k is a positive constant called the **force constant** or the **spring constant** of the spring.

Kinetic Energy and the Work–Kinetic Energy Theorem

$$W_{ext} = K_f - K_i = \Delta K$$

When work is done on a system and the only change in the system is in its speed, the net work done on the system equals the change in kinetic energy of the system.

Example 7.6 **A Block Pulled on a Frictionless Surface**

A 6.0-kg block initially at rest is pulled to the right along a frictionless, horizontal surface by a constant horizontal force of 12 N. Find the block's speed after it has moved 3.0 m.

$$W_{ext} = K_f - K_i = \tfrac{1}{2}mv_f^2 - 0 = \tfrac{1}{2}mv_f^2$$

$$v_f = \sqrt{\frac{2W_{ext}}{m}} = \sqrt{\frac{2F\Delta x}{m}}$$

$$v_f = \sqrt{\frac{2\ 12\text{ N}\ \ 3.0\text{ m}}{6.0\text{ kg}}} = 3.5\text{ m/s}$$

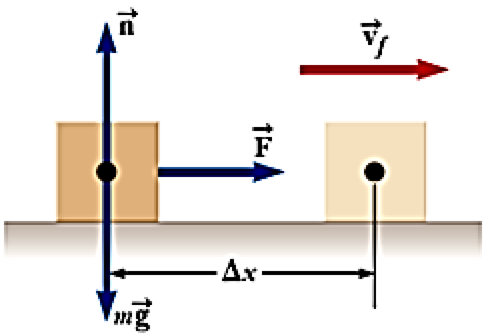


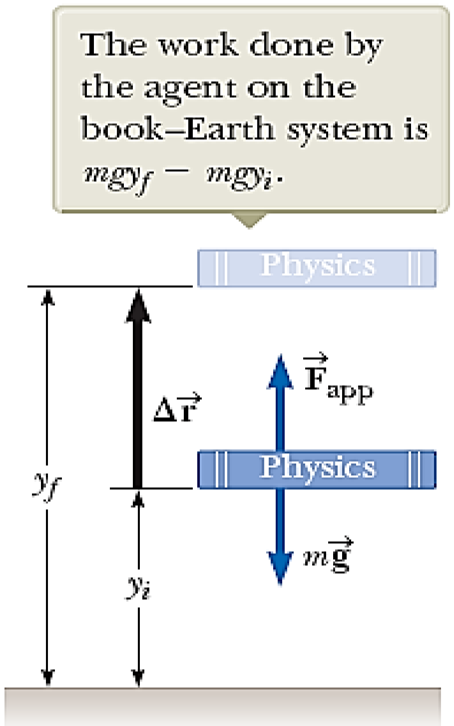
Figure 7.13 (Example 7.6) A block pulled to the right on a frictionless surface by a constant horizontal force.

Potential Energy of a System

$$W_{ext} = \vec{\mathbf{F}}_{app} \cdot \Delta\vec{\mathbf{r}} = mg\hat{\mathbf{j}} \cdot [\, y_f - y_i\, \hat{\mathbf{j}}] = mgy_f - mgy_i \qquad (7.18)$$

Therefore, we can identify the quantity mgy as the **gravitational potential energy** U_g:

$$U_g \equiv mgy \qquad (7.19)$$



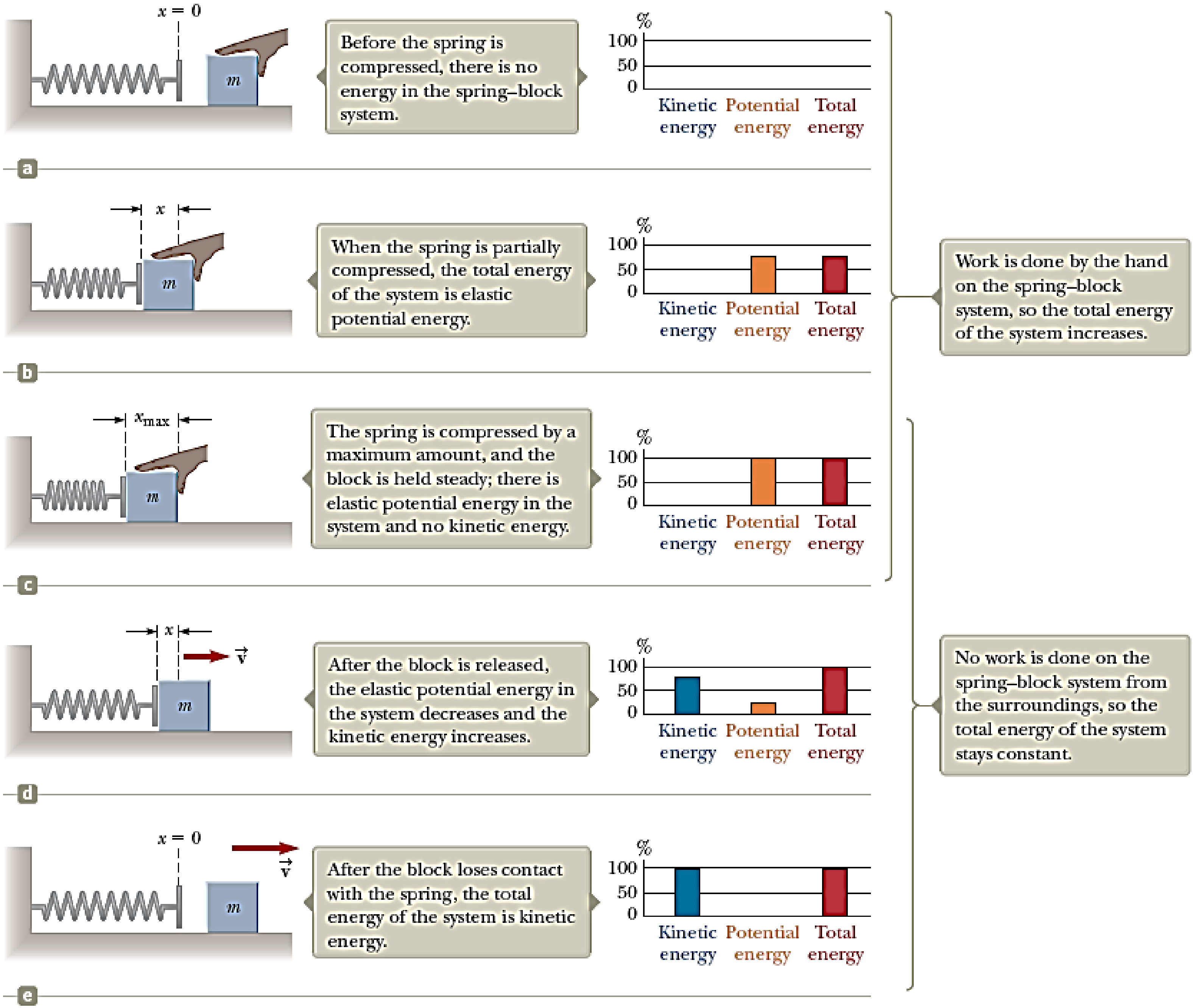
Elastic Potential Energy

The work done by an external applied force F_{app} on a system consisting of a block connected to the spring is given by the following equation:

$$W_{\text{app}} = \frac{1}{2}k{x_f}^2 - \frac{1}{2}k{x_i}^2$$

The elastic potential energy function associated with the block–spring system is defined by

$$U_s \equiv \frac{1}{2}kx^2$$



x = 0
m
Before the spring is compressed, there is no energy in the spring–block system.
%
100
50
0
Kinetic energy
Potential energy
Total energy
a
x
m
When the spring is partially compressed, the total energy of the system is elastic potential energy.
b
x_{max}
m
The spring is compressed by a maximum amount, and the block is held steady; there is elastic potential energy in the system and no kinetic energy.
c
x
$\vec{v}$
m
After the block is released, the elastic potential energy in the system decreases and the kinetic energy increases.
d
x = 0
$\vec{v}$
m
After the block loses contact with the spring, the total energy of the system is kinetic energy.
e
Work is done by the hand on the spring–block system, so the total energy of the system increases.
No work is done on the spring–block system from the surroundings, so the total energy of the system stays constant.

Conservative and Non conservative Forces

Conservative Forces

Conservative forces have these two equivalent properties:

1. The work done by a conservative force on a particle moving between any two points is independent of the path taken by the particle.

2. The work done by a conservative force on a particle moving through any closed path is zero. (A closed path is one for which the beginning point and the endpoint are identical.)

Non-conservative Forces

A force is non conservative if it does not satisfy properties 1 and 2 for conservative forces. We define the sum of the kinetic and potential energies of a system as the mechanical energy of the system:

$$E_{\text{mech}} \equiv K + U$$